Use of Game theory for conflict resolution in Spectrum negotiation between multiple RANs.

Muhammad Irfan

Submitted for the Degree of

Master of Science in Mobile and Satellite Communications

from the

University of Surrey

Department of Electronic Engineering

Faculty of Engineering and Physical Sciences

University of Surrey

Guildford, Surrey, GU2 7XH, UK

August 2009

Supervised by: Dr. A. Quddus

Prof. R. Tafazolli

DECLARATION OF ORIGINALITY

I confirm that the project dissertation I am submitting is entirely my own work and that any material used from other sources has been clearly identified and properly acknowledged and referenced. In submitting this final version of my report to the JISC anti-plagiarism software resource, I confirm that my work does not contravene the university regulations on plagiarism as described in the Student Handbook. In so doing I also acknowledge that I may be held to account for any particular instances of uncited work detected by the JISC anti-plagiarism software, or as may be found by the project examiner or project organiser. I also understand that if an allegation of plagiarism is upheld via an Academic Misconduct Hearing, then I may forfeit any credit for this module or a more sever penalty may be agreed.

Dissertation Title:
"Use of Game theory for conflict resolution in spectrum negotiation between multiple RANs".

Author Name
Muhammad Irfan

Author Signature -- Date: 14/08/2009

Supervisors name: Prof. Rahim Tafazolli
Dr. Atta ul Quddus

WORD COUNT

Number of Pages:	56
Number of Words:	13598

ABSTRACT

Rising demand of Radio Spectrum in wireless communications has made it a precious resource. It is not only due to the number of users increasing, but also due to the reason that more resources in terms of bandwidth are required for many advance services and developing technologies such as video telephony, mobile TV, wireless broadband access, wireless gaming etc. These increasing demands on bandwidth can be dealt with various modern techniques among which dynamic radio spectrum sharing is a prominent and powerful concept. Dynamic spectrum sharing in multiple operators of same or different kind of Radio Access Technologies (RAT) enhances its utilization.

Coexisting of multiple distributed operators on a common channel (Multiple Access) is a highly sought after goal. Game theory is an analytical tool to deal with a situation where each operator behaves in a selfish way to get access to as much bandwidth as possible. We investigate the impact of different strategies in non-cooperative multiple access game. Pure and mixed strategies are commonly adopted by users to utilize the spectrum opportunistically among themselves. In multiple access game, we investigate Nash Equilibrium games and best response functions to achieve in it in the presence of mixed strategy games. To optimise the utility, Pareto-optimality is introduced to find the desired equilibrium point by comparing the strategy profiles of different players of different interests in a game.

In game theory, Correlated Equilibrium works as a solution oriented technique with high performance (utility gain) as compared to the Nash Equilibrium. From Correlated Equilibrium two refinements are investigated with the help of linear programming, (i) maximum sum Correlated Equilibrium and (ii) max-min Correlated Equilibrium. An adaptive learning algorithm is used to find the convergence of joint probability distribution for both players with the passage of number of attempts. It is shown that given any arbitrary starting point of the game is played repetitively; the adaptive algorithm is able to converge to one of the pure strategy Nash Equilibrium of the game.

TABLE OF CONTENTS

LIST OF FIGURES

Acknowledgement

The writing of this dissertation has been one of the most significant academic challenges I have ever had to face. Without the support and guidance of Dr. Atta ul Quddus, this study would not have been completed. It is to him that I owe my deepest gratitude who undertook to act as my supervisor despite many other research and professional commitments. His wisdom, knowledge and commitment to the highest standards inspired and motivated me. I would also like to thank Prof. Rahim Tafazolli who gave me the opportunity to carry out this project.

LIST OF TABLES

Abbreviations

3G	3rd Generation
RAN	Radio Access Network
RAT	Radio Access Technology
NE	Nash Equilibrium
LTSA	Long Term Spectrum Assignment
STSA	Short Term Spectrum Assignment
INE	Individual Nash Equilibrium
JNE	Joint Nash Equilibrium
T	Transmit
Q	Quiet
CE	Correlated Equilibrium
BNEP	Best Nash Equilibrium Point
WNEP	Worst Nash Equilibrium Point
CE	Correlated Equilibrium

INTRODUCTION

In recent years, cellular communication has experienced an extraordinary growth, today there are billions of users around the world using the mobile phones to communicate with each other. With the growing importance of wireless applications and development of many new advanced wireless technologies, the radio spectrum has become a most precious resource in the entire world. To overcome this problem, government organizations throughout the world manage the assignment of radio spectrum within their countries. But within an organization the parameters involved in the management of radio resource are modulations schemes, transmit power, error coding schemes, channel allocation etc.

Many studies and observations have shown that a considerable portion of spectrum is always not in use at different locations. According to [1], about 62% of permanently allocated spectrum is unutilized in most of the areas; and these areas are called white spaces. For wireless service operators, it worth billions of pounds, to save this large amount of money, an operator can sit there silently and can access the spectrum opportunistically. Larger white spaces point towards the dynamic or opportunistic spectrum access which can significantly alleviate the spectrum shortage.

1.1 Background and Context

Radio resource management in wireless communications is becoming more efficient with the help of cognitive radio techniques where the devices/nodes change their parameters for the interference free communication. In fact, there are different factors involved in the change of these parameters for transmission and reception such as user behaviour, frequency, power control and state of the network. Cognitive radio can be categorised with different ways such as (i) full cognitive radio where the parameters taken into account are observed by network node. (ii) Spectrum sensing cognitive radio where only radio frequency is considered. Licensed and Unlicensed band cognitive radio spectrums are other categories.

Mean time we study the game theory, it was originally developed to examine the competitions where someone produces better results on the expenses of someone else. The main idea behind the game theory is rationality which is based on the thoughts and assumptions where everyone tries to maximise his profit, reward or subjective benefits. Not surprisingly, some applications of game theory have been applied in wireless communication, mainly focused on radio resource management in competitive environments. In non-cooperative game theory, players use different strategies to raise their payoffs. In wireless communications, payoff increases only by using the spectrum as more as possible by adopting different strategies. Strategies can be of pure or mixed. In pure strategies, players deterministically decide about their moves, which mean players clearly know about their decision. And in mixed strategies, players have probability distribution over their pure strategies [2]. In general, mixed strategies are used in conflicting situations where players are not sure about the strategies which can be played by their opponents.

1.2 Scope and Objectives

The main objective of this project is to optimise the utilization of radio spectrum. At the moment, in fixed spectrum allocations, multi-year licenses of radio spectrum are granted to different organizations depending on their services, where its utilization becomes limited to a few operators and services. In dynamic sharing of spectrum, it can be utilized very actively, users can switch their frequencies very quickly and with in very short interval of time. Our key objectives are to investigate are as:

- Gain the understanding of basic and some advanced concepts of non-cooperative game theory.
- To understand the use of game theory in wireless communications, additionally the use of Multiple Access application.
- To investigate different approaches of game theory with which we can maximize our utility, pure and mixed strategy Nash Equilibria are investigated.
- To investigate the concept of Correlated Equilibrium, it will enhance our utility slightly.
- Develop the simulation software to represent our results. Evaluate the simulations results and compare them with calculated values.

1.3 Achievements

The outcomes of the project in selfish and non-cooperative environment, where all players try to get access to the maximum part of spectrum without cooperating with other players which accelerate the collision probability. High collision rate results in the form of data packet and power loss which actually leads to inefficient usage of radio spectrum. In order to make the usage of radio spectrum more efficient, a game theoretic approach is investigated where players find individual Nash Equilibria at Base Station (BS) level. In Multiple Access game, we find two pure strategy Nash Equilibria (T, Q) and (Q, T), these are the only two states for both players to get the maximum utility using pure strategies. By adopting mixed strategies, best response function is called for both players to achieve the maximum utility.

To optimise the utility, Pareto-superior and Pareto-optimality approached are investigated to find the desired equilibrium point by making a comparison of different strategy profiles of a number of players of conflicting interests in a game. Correlated Equilibrium works as a solution oriented technique with high performance along with Nash Equilibrium. From Correlated Equilibrium two refinements are investigated with the help of linear programming, (i) maximum sum Correlated Equilibrium and (ii) max-min Correlated Equilibrium. No-regret learning algorithm is used to find the convergence in joint probability distribution for both players with the passage of number of attempts. Cooperative game theory and the parameters of Cost function can be investigated in future for the efficient utilization of radio spectrum.

1.4 Overview of Dissertation

The report is organised into five main chapters:

Chapter 1 introduces the project and discuss what its scope is? And what are the objectives behind it? It also signifies the achievements of the project.

Chapter 2 introduces the basic notions of the game theory. In fact, it is an analytical approach to deal with radio resource management in wireless communication. A game actually formalized with a set of parameters in non-cooperative manners where players make one move as their strategy and that strategy does not have any dependency on others. The probability of making the move is assigned zero to all strategies except to one and that strategy is named as pure strategy. Different tactics are used to solve the problem such as to find the strategy which dominates iteratively. Nash Equilibrium finds one of the best solutions in the multiple access game using the pure and mixed strategies for two or more than two players on a common channel. Optimal utility is determined by the Pareto optimality approach where we compare different strategy profiles and find the most suitable one. We also discuss the pros and cons of game theory in wireless networks. Four different scenarios (Forwarder’s dilemma, Joint packet forwarding, Multiple Access and Jamming) are considered for better understanding and on-ground implementation of game theory in wireless communication.

Chapter 3 explains the model of the project. We discuss the problem statement with physical view. In normal situation, operators use their own dedicated spectrum. But when they face high data rate demands from their users, both operators try to access the FSS spectrum opportunistically. To share the FSS spectrum in between multiple operators, we use Multiple Access scheme under the influence of game theory approach. In game theory, both operators apply different strategies to maximise their own utilities. We find two pure strategy Nash Equilibria (T, Q), (Q, T) and one mixed strategy Nash Equilibria using the best response of both operators. In this scenario, we assume both of the operators as the secondary users and they compete for the FSS spectrum which is originally owned by the Fixed satellite service and it is assumed as the primary users. Mixed strategy individual and joint Nash Equilibria are derived as well.

Chapter 4 represent the investigation of multiple access game with Correlated Equilibrium. Interference and channel availability matrices are discussed in section 4.2. Every secondary user knows about the available channels and also about the users who can interfere with its communication, it adopts different strategies, which actually hold the information and tells whether it can transmit or not? And if it can transmit then what is the suitable rate (data rate)? From previous chapter, we know the best utility of users, but it is possible to improve this utility a bit more by using the concept of Correlated Equilibrium. The example discussed, shows that there is 6.67 % improvement in the utility (calculated not simulated yet), which is one of the important goal to achieve in this project. No regret learning algorithm is investigated to find the convergence in joint probability distribution for both players with the passage of number of attempts.

Chapter 5 gives the conclusion of our project, the simulation results and calculations are analysed here. Brief information is given about the work which can be carried out in future.

2 LITERATURE REVIEW

2.1 Introduction

In wireless communication, during radio resource management we do need to solve a number of practical problems, for those purpose it is essential to analyze the situation and possible outcomes. To analyze such kind of situations, a mathematical and statistical approach comes into practice which is called game theory. Usually game theory is used in conflicting situations and its main objective is to analyze and elaborate the behaviour of rational players in non-cooperative games. Basically in wireless communication it is an analytical tool for dealing with various problems. We review the literature of game theory in depth in the following sections.

2.2 Game theory:

As we have discussed above that game theory is an analytical tool to solve the problems in conflicting situations. It is actually very vibrant and mounting field; it can also be defined as the field of mathematical scenarios of variance and the mutual aid among the rational and very intelligent players. It provides the mathematical tools for optimizing and analyzing any circumstances, where multiple decision makers make their decisions that affect each other's interests. Game theorists struggle to recognize the root of cooperation and conflict by consulting the quantitative scenarios and hypothetical examples. Assumption-based examples may be very simple as compare to the real life problems but this simplicity may work as the base to understand the solution of the very complicated problems [14]. In fact, any kind of inquiry can be resolved by following the simple procedure and take the problem from its scratch.

2.2.1 General Notions:

2.2.1.1 Game

A word game, as we use it everyday. Usually we say football is more difficult game than the rugby, we mentioned here *a word game, which actually tells us the rules or some technical aspects of that action* [15]. Here we will use another word "move", it shows a point in a game that which tells about the selection from available options? There are huge amount of games with respect to their number, or variety, or strategy. First we go ahead according to the numbers. One person game, two person game, for example Solitaire is a one person game and snooker is a two person game. We can say *n* person game, in reality it does not mean that there must be *n* player in the game otherwise you can not play it. No, actually it means, the rules of the game allow that *n* persons or players (the notion player discussed later) can fall into the game. Games can be flexible such as chess is a two person game, only two people can participate in it, but we can enhance it by making two teams of three players each. In fact it is still two person game but with six players. With a set of player we can have more games such as rectangular games, one person ZM (zero sum) game, rectangular games with saddle points,

2.2.1.2 Player

Usually a game refers to a situation where one or more than one player is involved. And *the participants involved in the game are supposed as the players* [15]. We know that a game always revolve around the player which also plays very important role in decision making. Most of the times every player hold some resources, also keep available some alternative actions such as to collaborate or take some suggestions or communicate with helpers or at last some inherent interests like utility concerns. For example, in any economic game there are some set of rules such as information channels, technology, distribution laws and all of these features actually help the decision maker or the player to chose his best.

Outside of the game, all the players are alike. Game theory does not distinguish them in their own premises. But in all of our discussions we assume that our players are rational, they are very conscious and have clearly defined their goals. They will have penalty of options of choice, they do practice of them but within the limits. The option of choice means the freedom of choice. In any conflicting situation which is taken from the practical scenario is always very complicated and it is also very difficult to analyze the outcomes is the presence of many other factors. Mathematically, we can a make *simplified model depending on the situation; such model is called a game* [14][15]. A game, actually formalized with a set of rules, we know that a spirit of competition is a part of the game and at the end victory goes to one or to other.

2.2.1.3 Strategy

The term *strategy* in game theory is used to denote *the selection of a probability distribution over events and the subsequent use of this distribution at each trial in a series to determine the particular succession of choices to be made* [15]. If we assume that, we have two choices A and B. The proportion of choices of each of the two alternatives is equal to, or matches, the probabilities with which the two events occur. It is curious that selection should come to adopt a strategy, it is understood that we will do our best to predict correctly which of the two events will occur on each trial. A pure strategy in the above situation would be to choose the A event on all the trials. Thus the pure strategy $P_A = 1.00$ and $P_{\mathbf{B}} = 0.00$. A mixed strategy in contrast involves distribution choices between the alternative events, as $P_A = 0.80$ and $P_B = 0.20$, or in $P_A = 0.85$ and $P_B = 0.15$.

Different strategies vary with respect to their outcomes. The pure strategy of invariably choosing the more frequently occurring event may be expected to yield more correct choices than a mixed strategy. With respect to the variability, however, a mixed strategy is less boring to execute than a pure one, but the expected utility of the choices situation is thus determined in part by the strategy employed in it.

2.2.1.4 Utility

The term utility might be used interchangeably with the term subjective value. An other term "intrinsic worth" has also been suggested (oxford dictionary). *The utility of an outcome is the value of that outcome to an individual; we may say it as the benefit of a correct response* [15], alluding to some index of the subjective value which a particular individual attaches to responding correctly. The utility of any individual can come from different directions and the overall utility of any possible outcome may depend on the subjective value of each of several conceptually distinct aspects of that outcome. To predict choice behaviour, one must identify the various aspects or components or factors of the situation to which positive or negative utility is associated. With information about the utility each component of each outcome, it is possible to assess the utility associated with any particular strategy. Such an analysis provides a rationale for experimental operations.

2.2.1.5 Utility Parameters

We have two basic utility parameters; the first one is the utility of a correct choice. An individual attaches some value to making a choice which is subsequently confirmed by the occurrence of the predicted event. He will place even more value on a correct choice if he receives some payoff for correctness. Further, the utility of a correct choice will vary directly with the utility of that payoff; the utility of a correct choice will be greater. And the second one is the utility of variability. In this case, an individual will keep in mind the negative utility as well as the positive because it has some utility of avoiding monotony, of maintaining variability.

2.2.1.6 Game theory with perfect information

Now we turn towards another kind of game, is the game with perfect information. This kind of game are usually characterized by the fact that at each and every point the player whose turn it is to move knows exactly what choices have been made previously. Now the games with perfect call can also be defined as the point where each of the players always keep remember each and every thing they did in past, or knew their previous moves or actions. Therefore, every two person game which can be played by two players rather than two teams (discussed earlier) is a game with perfect recall.

The notion of the game with perfect recall can be made precise in terms of the information sets of the game; we assume here a game with perfect recall is the one which fulfil the following condition. Let P and Q be any two moves, both of which are made by the same player and such that P precedes Q in some play of the game. let U and V be the information sets containing P and Q respectively, and suppose that each point of U presents k alternatives, let U_i (for $i = 1,2,\ldots k$) be the set of all vertices of the tree which can be reached by taking the *i*th alternatives at some point of U, then for some *I* we have V subset of U_i Non-cooperative static games:

Non-cooperative static games can be defined as where the players make one move as the strategy and that strategy does not depend on any other. If we define here a game, which actually comprises on different players(in this section we consider only two players in our game, but generally there can be any number of players), p_1, $p_2 \in P$, where p denotes the player and its subscript denotes the player number [2], for our convenience we set subscript *-i* for all of the player belongs to set P other than *i*. all these players (*-i*) are considered as the opponents of *i*. S_i represents the strategy set for player *i*. strategies can be of two kinds, (i) pure strategy, where the probability of making the moves is assigned zero to all strategies except to one strategy (then this strategy clearly makes the move). (ii) mixed strategy, where different strategies are chosen with different probabilities. Third part of the game is the utility, which is actually a whole benefit from the game. The utility set can be represented as $U = \{u_1(s), u_2(s)\}$, U shows the total utility of players p_1 and p_2 as $u_1(s)$ and $u_2(s)$ respectively using strategy s.

2.2.2 Game theory in wireless communications

The approach of game theory in wireless communications was not used in the early stages of technology development but now a number of its applications are extensively worn in wireless networks. In [11] cellular systems, the management of the power control system was based on the utility. And then in [13], the management of the power control system was cost based in wireless networks, where the concept of non-cooperative game is applied and confirm that the inefficient Nash Equilibria can be improved by implementing the cost or pricing strategy. In downlink radio resource management Berry and Marbach [12] also implement the pricing approach but they uses two schemes (i) the scenario where BS stations have the information about the utilities of the players, (ii) and in second scenario the BS hold nothing related to the utility. The decision makers are believed to be the devices which want to communicate or transmit or receive the packets to and from each other. Before going into more details and examples of game theory applications in wireless networks or communication, we study some advantages or why do we use game theory in wireless communications.

2.2.3 Advantages

In wireless communication, we have a number of advantages of game theory [16], which are as:

2.2.3.1 Local Information

All the users who participate in any game, they keep checking the outputs from the game, and then decide whether it is beneficial for them to inject more resources into the game or not. And for that purpose it is not required to collect the whole information and implement more constraints in a centralized manner. Because, the local information can fulfils all the necessary requirements regarding the optimized utility.

2.2.3.2 Robust outcome

We know, for optimized results we need absolutely correct information. If we do not have the exact information about our strategies which we are going to play and possible outcomes from them, then we can not obtain good favourable results. It is quite sure that the local information is always correct and perfect, therefore, the utility from such kind (distributed) of strategies is always more robust than.

2.2.3.3 Combinatorial nature

Different approaches are used in daily life for the optimization of the results such as programming, it is very difficult to tackle and over come combinatorial problems. But with the help of game theory, I mean by the way we deal with the problems in game theory such as converting the problem into strategic form and discuss the issues. Depending on the problems, for example: we can adjust the channel coding rates according to the requirements or modulation level can be readjusted. Therefore, we can say the game theory is more suitable or convenient to analyse the problem.

2.2.3.4 Rich mathematics

Outcome of complicated games depend of number of factor, to sort out those complications we use different tools of mathematics, most importantly when all the players behave non-cooperatively with each other and try to maximise their own utilities. Dynamic game theory is employed when players play different strategies at multiple times in a game. Therefore for the best optimization mathematical tools play very important roles [16].

We take here a few applications of the game theory in wireless communication in the form of examples

Example 2.1. Forwarder's Dilemma:

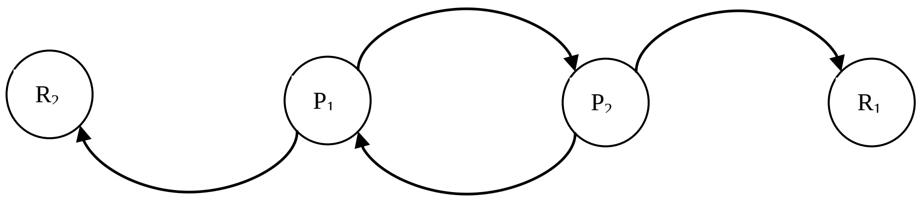

Figure 1. Forwarder's Dilemma Game [2]

In this example, we assume that we have two players P_1 and P_2, both of them wanted to send their data packets to receivers R_1 and R_2 respectively. In this scenario, the communication is possible only if each of them forwards the data packets of each other. If they forward the packets, they will get its reward, conversely if they do not cooperate with each other and drop the packets, they will get a reward of nothing. In strategic form [2], this game can be represented as follow.

P_1\P_2	F	D
F	(1-c, 1-c)	(-c,1)
D	(1, -c)	(0,0)

Table 2.1: Forwarder's dilemma in strategic form.

In above table, players P_1 and P_2, both will get a reward of 1-c each if they forward each other's data-packets, conversely in non-cooperative scenario if each of them drop the packets they will get a reward 0. In case 2 and 3, one of the both players is dropping the packet while other is forwarding and due to this reason it gets a reward 1but other one gets –c.

Example 2.2. Joint Packet Forwarding:

In this example, we have one sender and one of its receivers represented as snd and rsv respectively in the figure.2. Sender has some data packets, which he wants to send his receiver rsv. It is clear from the figure that snd can not reach directly to its receiver, it needs some help from the player P_1 and P_2 to work as forwarders for its successful transmission.

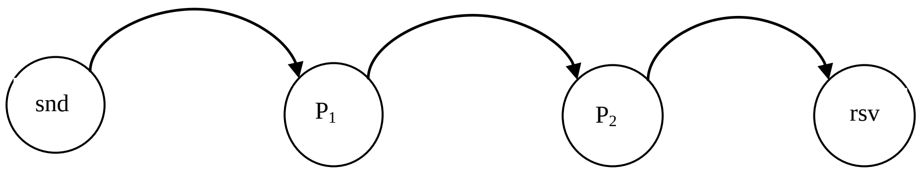

Figure 2. Joint Packet Forwarding [2]

As discussed in example 2.1, the processing cost (here the forwarding cost) is very low, as $0 < C << 1$. If both of the players contribute in transmission effort and forward the data packets, they will get a reward of 1 from any of the transmitter or receiver. But if any of them drop the packet, the link will break down and transmission will become unsuccessful.

$P_1 \backslash P_2$	F	D
F	(1-c, 1-c)	(-c,0)
D	(0, 0)	(0,0)

Table 2.2: Joint Packet Forwarding in strategic form [2]

Table 2.1 shows the strategic form of the Joint packet forwarding scenario, where if both intermediate players forward the data packets of sender to the receiver, then player P_1 and P_2 will have a utility of 1-C and 1-C respectively. In second option, if player P_1 forward the data packet but player P_2 drops it, transmission will become unsuccessful and it will cost –C to the player P_1. In third and fourth options when player P_1 does not forward the data packet and drops the data packet, it will never reach to the player P_2, and then in this case both of the players will receive no payoff because they did not make effort to forward the data packet.

<u>Example 2.3. Jamming:</u>

In jamming game, we assume that both players P_1 and P_2 have full access to the channel and want to transmit their data packets on each time slot towards their destinations. Due to the high collision probability we split the channel into two parts under the rules of Frequency Division Multiple Access (FDMA). Now for the high throughput player P_2 will try to access the channel of player P_1 as well as its own channel. The main aim of P_2 is to interrupt the player P_1 and use that channel by self on mean time, this approach is assumed as jamming in wireless communication. Now it is very clear, that challenge player P_1 will a face to successfully transmit its data packets in the presence of player P_2 (jammer). If it successfully transmits, it will get a payoff of 1 and will get a payoff of -1 if the player P_2 jams its data packet before reaching on destination.

P_1(sender)\P_2(jammer)	Ch_1	Ch_2
Ch_1	(-1, 1)	(1, -1)
Ch_2	(1, -1)	(-1, 1)

Table 2.3: Jamming in strategic form [2]

Strategic form of the jamming game shows all the possible outcomes from both players on both channels. There is no pure strategy Nash Equilibria (will be discuss later) in jamming game. it is also clear, that there is no solution via iterated dominance (discussed in next section) as well.

2.2.4 Iterated Dominance:

It is very simple and interesting to solve such a situation which is expressed in a proper organised form. The solution of the game actually let us knows very clearly that what can be the strategies of each player in the game. We assume, our players in game are very rational then strict dominance is the best possible solution. Strictly dominated strategy can be defined as if the utility of player *i* is always smaller by using the strategy s_i' instead of s_i [2]. As by considering the example in table 2.1, game can be solved using iterated strict dominance. We remove the strictly dominated strategies from the table and get the possible solution. As we know that player P_1 is rational then suppose that its F strategy is strictly dominated by D which actually removes the very first row from the table. But with the player P_2's concerns, it will remove very first column. We see the result is (D,D) and the utility is (0,0) .

2.2.5 Nash Equilibrium:

Some times it becomes very complicated to solve every game by using the iterated dominance scheme. We consider here another example, Multiple Access game. In this game multiple players try to access the single channel for their data transmission. We assume again that we have two players P_1 and P_2 in our game who wants to send their data packets on each time slot to their receivers R_1 and R_2 respectively.

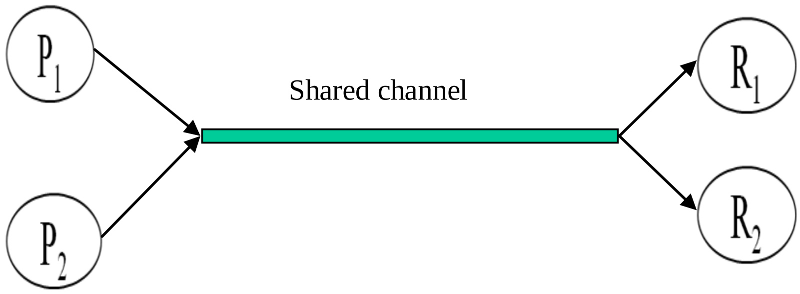

Figure 3. Multiple Access Game

As we know those players are sharing the same channel for their transmission and they are with in the power range of each other. Now if any of the players transmits its packet with a cost of $0< c << 1$, the packet can reach successfully to its destination if and only if the other player keeps quiet, mean time if second player transmits as well then collision will occur and the packets will lost. Multiple access game can be represented in strategic form as:

$P_1 \backslash P_2$	T	Q
T	(-c, -c)	(1-c, 0)
Q	(0, 1-c)	(0, 0)

Table 2.4: Multiple access game in strategic form [2]

From the above table, we understand that no dominated strategy exists here in the game. Therefore, we use another concept which is called best response. If any of the players transmits its data packet then the best response of the other player is to keep quiet. Conversely, if the second player transmits then it is the responsibility of the first player to keep quiet. Therefore, we can say that there two strategy profiles exist in the game (Q, T) and (T, Q). to fined out such kind of strategy profiles Nash introduced in [3], a phenomenon of Nash equilibrium. Nash equilibrium means that it is not possible for any player to change its strategy unilaterally to increase its utility [2][3].

2.2.6 Mixed strategies:

As we have already stated, that *the probability distribution chosen over pure strategies makes them mixed strategies.* Mixed strategy profile is constructed by the probability distribution over the pure strategies of each player. Opponent's strategy profile is denoted by σ_{-1} in pure strategy profiles. For every player *i* we have a utility to profile σ is given as:

$$u_i(\sigma) = \sum \sigma_i(s_i) u_i(s_i, \sigma_{-i}) \tag{2.1}$$

Where $s_i \in S_i$.

2.2.7 Pareto-optimality:

Up to now, we have seen pure strategy, mixed strategy, Nash equilibrium and best response. We know that in any game there might be one or more than one Nash Equilibria. We study here another method through which we can identify the equilibrium point in a game. Pareto-optimality is a concept of comparing strategy profiles. First we define the concept of Pareto-superiority, *any strategy profile s in a certain game is suppose to be Pareto-superior if and only if it produces the larger utility instead of using strategy s'.* Mathematically it can be represented as

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s'_{-i}) \tag{2.2}$$

Pareto-superior strategy can also be expressed as, the strategy profile s can be Pareto-superior if the utility or payoff of contestant *i* is increased by altering its strategy *s'* to *s* without decreasing the utility or payoff of any other player in the game. thus, the strategy profile s' is said to be the Pareto-inferior as compare to the s. according to the concepts we have discussed above, we can find the best strategy profile. Now we can define the Pareto-optimal strategy, *" A strategy profile s^{po} is Pareto-optimal if there is no strategy profile which can be Pareto-superior to s^{po}"*[2].

2.2.8 Single Stage Game:

We know iterated or repeated game which is actually an extensive form of the game where actions are repeated extensively. These repetitions are of the base stage game. In single stage base game, both players select their actions once and for all. In this kind of game both players in fact consider the impact of their own actions on the other player's actions in future. Nash equilibrium is such games are always unique and maximum Nash equilibrium is achieved with appropriated strategies.

2.2.9 Dynamic Games

When players play the single stage game repeatedly, then the single stage game becomes the dynamic game. When they interact with each other more than once, they can keep some information with them about the strategies of their opponent players. And the strategies played after that depend on each other to get the maximum utility. Some times players cooperate with each other for the mutual benefits but in non-cooperative games, it does not happen. Here we will discuss about the information which can be useful for the time being for some players and some characteristics of using the dynamic games in wireless communications [16].

Before going into the depth, we consider here some basic definitions of the dynamic games. Such as, sequential game: sequential games are those where players play their strategies according to predefined order. In this kind of games, some user can easily understand the strategies of the users who preceded them. But if none of the users monitor the strategies then automatically game becomes a simultaneous. As discussed above in section (2.2.1.6), if players share all of the factors involved in the game with each other and they keep them in their knowledge then the game should supposed to be with the complete information. In any player knows exactly about the strategies or moves of the other player, for example, if this is time for my move or strategy and I hold the whole information about the strategies of the opponent player that should be the complete information game. if I do not exact information or some strategies information is missing that should be supposed as the imperfect information game.

An extensive form of the games also called as the game tree. It actually represents the sequential games in the graphical form.

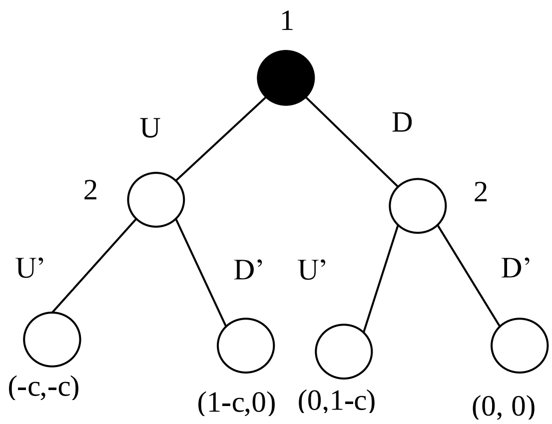

Figure 4. Extensive Game form [2]

This representation of dynamic games holds the whole information about the strategies, their corresponding players, the outputs and their sequential order. The extensive form of the game usually comprises on a set of nodes, nodes are those points where player select the strategies or moves [16]. All the nodes are connected with the vertices, which show what are the strategies or moves, those can take place. At the start of the game, a very initial node which is called root node, takes the first action in game. At the end of the game, each terminating node hold the possible output of the game if it finish on that node

2.2.10 Auction theory

Another branch of game theory is the Auction Theory (AT), which actually tells us, what properties of game theory can be applied in the auction markets? Very similar comparisons take place in auction theory such as revenue comparisons based on equilibrium strategies as in game theory [21]. Different types of auction are used conventionally such as first-price sealed-bid auction where the participants in bid submit their bids in sealed envelop to the auctioneer and the highest bidder wins the bid. Any ways, this is not our point of concern; our focus is the game-theoretic auction model. In reality, it is a mathematical representation of players with their strategies to play and finally the utility or payoff. Buyers and sellers are considered as the players and bid functions are set as their actions. Generally strategic bidding has two major categories [21],

- Private value model: In this kind of model, each player or participant think that the participants of the game are selecting a random private value from a probability distribution.
- Common value model: in this kind of model, every one considers that the players are getting the arbitrary message from a probability distribution common to all bidders.

Auction theory has proved itself as a fantastic approach of dealing radio spectrum management in recent years.

2.2.11 Pricing theory

Pricing theory in any field of life revolve around the asses pricing. In any business *“price is the result of trade we assign a numerical monetary value to a good service or asset*”[21]. Price is just like information which gives a chance to understand the value of product. Our main objective is to review the literature which tells us the role of price in the management of radio resources. We know the efficient use of radio spectrum is very big challenge. Enforcing different pricing strategies in all sectors of spectrum management can make it very efficient. Such as pricing with respect to the power control, efficient power control algorithm [22]can save the energy which is directly linked to the price. Pricing functions can implemented to the transmit power, distributed implementation of the pricing functions makes it very simple because it can be broadcast or transmit from the base station to terminals [22].

2.2.12 Correlated Equilibrium

The conception of correlated equilibrium was introduced by Aumann in about 70s. The fundamentals of this phenomenon are as, before execution of the game, every participant of the game has some instruction such as a private message [18]. That private message helps that particular user in choosing or making his decision for his action. Aumann’s approach of correlated equilibrium of a game is very similar to the Nash equilibrium but addition is private message or signal. All the privately received messages take part in the generation of all the Correlated Equilibria. The Nash equilibrium and correlated equilibrium divides their ways as, if the privately received messages or signals for a player does not depend on the signals received by any other player, it means that it produces the Nash equilibrium of game which can be under the pure or mixed strategies. But there can be a possibility that the privately received messages are correlated with each other, in that case the Equilibria achieved from those strategies selected by the correlated messages can be different [18]. In fact, we hold a large number of actions and correlated equilibrium acts as the probability distribution on all of those actions. From all of the private instructions received for the selection of the actions, the player understands that, the received message gives a good response as compare to the rough actions of the opponent players [10][18].

The correlation of the Equilibria comes from the measurement of the regret which is always based on the some expectations of the utility from the past periods. All the players have some information about their history of actions along with the utilities achieved from those actions. All these activities go through certain procedures at each occasion, the player can either keep similar strategy as it used previously or can select some other, if it chooses new strategy and get a smaller payoff then it would not have any regret of using the strategy it had in past. But if the situation shows the result inverse, then there should be some regret of having that strategy.

2.3 Summary

In this chapter, we have discussed the basics of the game theory, additionally how does it work? And how it can be used in the environment where wireless devices work as the players. We explained the basic notions such as game, player, utility and strategy etc. For the use of game theory in wireless communication or networks we discussed four different scenarios. In each of the scenario, we studied one example which actually explains the real time implementation of the game theory in wireless communication. Our main focus was the non-cooperative behaviour of all users in game theory. In our considerations, it is supposed that all of the devices as our players, and expected results were based on the same assumptions. Their results can be different if we replace the players with some one else, other than devices such as network operators. Additionally, there is another branch of game theory which is cooperative game theory, we did not go through that one in detail, but it can be used in centralized environment where additional signalling is not a very big issue. I think this basic review of literature will help us in solving our problem in next chapter.

3 APPLICATION OF MULTIPLE ACCESS GAME TO TWO RAN SHARING A COMMON BAND

3.1 Introduction

As we have discussed in chapter1 that wireless communication has experienced an extraordinary growth in recent years, and radio spectrum is considered as the most valuable resource. To control this situation, different organizations have conducted a number of research studies and found that about 62% of permanently allocated spectrum goes into waste when no one access it [1][17]. To utilize the unutilized radio spectrum, there have been different proposals but one of them is discussed here, access the unused spectrum opportunistically by distributed users. To measure the payoff of utilizing the unused spectrum, we follow the application of game theory called multiple access scheme, where multiple operators/players share the same portion of spectrum and transmit their data packets on it. All the users try to access as more spectrum as possible for long period of times to maximise their utilities. To optimise the utility of all players, pure and mixed strategy Nash Equilibria will be investigated here.

3.2 LT Spectrum Assignment

For Long Term (LT) spectrum assignment to any operator, state authorities specify the criteria, different policies depending on services and consider all technical and non-technical issues before spectrum assignment permanently for long term to any operator. In LT spectrum assignment, operators (Radio Access Networks) can use the same kind of Radio Access Technology (RAT).

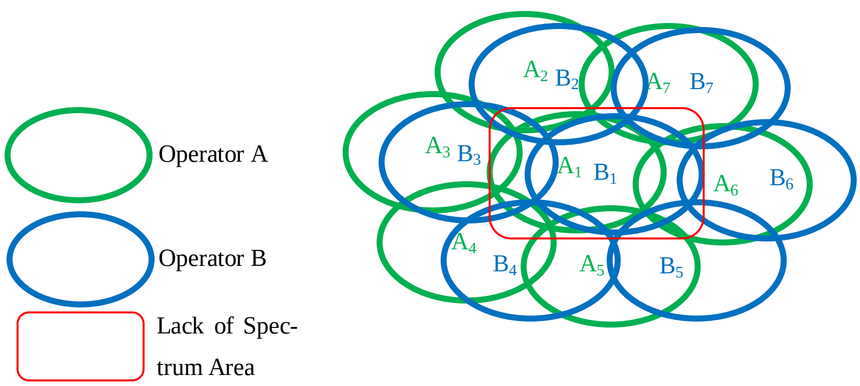

Figure 5. Cell Architecture

It is assumed that, we have two operators A (RAN 1) and B (RAN 2) in our scenario and both are using the same Radio Access Technology. They have got their own dedicated bands (LT spectrum) assigned to them, operator *A* has a band of 14.6 MHz (from 1920.3 to 1934.9 MHz) for Uplink and 14.6 MHz (from 2110.3 to 2124.9 MHz) for Downlink, Operator *B* has 10MHz band (from 1934.9 to 1944.9 MHz) for Uplink and 10MHz (from 2124.9 to 2134.9MHz) for Downlink as shown in figure-6. In normal operation there is no intersystem interference between users of operator A and operator B.

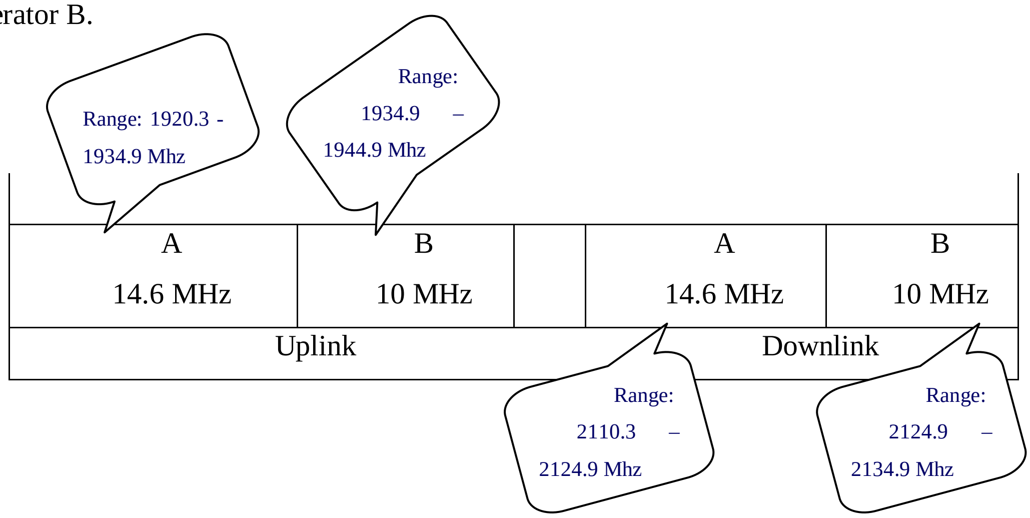

Figure 6. Spectrum Allocation

3.2.1 Problem statement

Up to now both of the operators were working very smooth and nice but due to the high traffic demands, they face spectrum shortage. Due to the lack of resources they can not fulfil the needs of their subscribers, because of this reason they look forward for more spectrums. It is known, that the base stations of both operators have installed the spectrum agile devices[4] which can sense the unused spectrum of any other frequency. It is assumed that there is some unused FSS spectrum in same area, its range is shown in the following figures, which can be accessed by both operators.

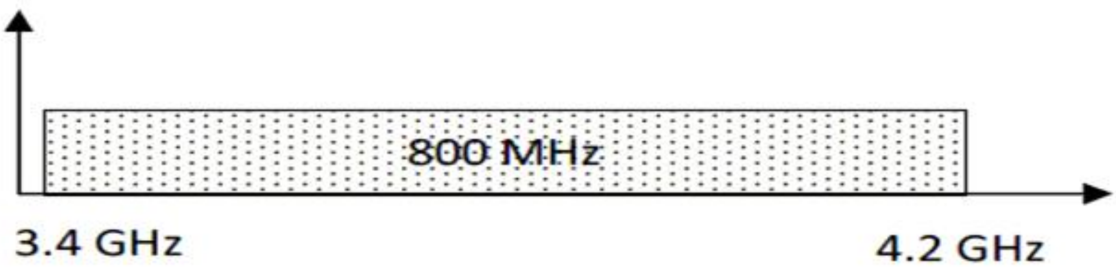

Figure 7. FSS Spectrum

Operators face a lack of spectrum problem in Red area as shown in figure 5, where cell A_1 and B_1 overlap with each other. There have some unused carriers in FSS spectrum, which can be accessed by both cells using the Multiple Access technique.

3.3 Revisiting Multiple Access Game

Now we study the solution model of the project and its behaviour for possible outputs. In model, three major parties are involved, primary users, secondary users and channels (i-e radio resources). Primary users are the equitable owners (FSS) of the radio channels and they hold the priority of its use. Secondary users are those, who can access the spectrum if it is not in use of primary user, in fact secondary users are equipped with spectrum agile devices [4] which make them able to know about the environment and let them know whether the primary user is using its channels or not. When primary users become idle then they utilise or share the channels for their transmissions.

From the above discussion, it is very clear that the channel availability is possible if primary users have nothing to transmit. Additionally, secondary users can jump into field to compete for the unused channel with each other. If multiple secondary users transmit their data packets at same time on same channel, then collision can occur. The penalties, due to collision are in the form of power waste and packet loss.

3.3.1 Finding Nash Equilibrium in Multiple Access game

Here, we study that how can it find the Nash Equilibrium in *multiple access scheme* using game theory, in next chapter we will investigate the effect of correlated equilibrium on overall utility and optimum utility using the linear programming method on multiple access scheme and at the end, we will go for the No-regret algorithm for the learning curves of utility.

$P_1 \backslash P_2$	T	Q
T	(-c, -c)	(1-c, 0)
Q	(0, 1-c)	(0, 0)

Table 3.1: Multiple access game in strategic form

In strategic form of the Multiple access game, there are two possible actions T (Transmit) and Q(Quiet). If both player P_1 and P_2 transmit their data packets at the same time, collision will occur and that will cost them lose of data and lose of energy. But in second and third options where only one player transmits at one time, there is no collision and both of them getting a reward of (1 - c).

In last option none of the players transmit, therefore no one get any kind of reward. Now we define the Nash Equilibrium, by definition it is a technique which tells us, *by changing the strategy unilaterally none of the user can maximise its own utility unless by changing the strategy of his opponent user*[2].

$$\mathrm{U}_i(r_i^*, r_{-i}) \geq U_i(r_i', r_{-i}), \forall s, \forall r_i' \in \Omega i \quad (3.1)$$

3.3.1.1 Mixed strategy individual Nash Equilibria

We assume that, player P_1 decides to transmit with the probability of x. and y is the probability with which P_2 make its decision of transmission. Conversely 1-x and 1-y are the probabilities of being quiet of player P_1 and P_2 respectively.

Solution 1:

We know $\quad u_i(\sigma) = \sum \sigma_i(s_i) u_i(s_i, \sigma_{-i})$

Then

$$\mathrm{U_{p1}} = (\mathrm{x})(-\mathrm{c})(\mathrm{y}) + (\mathrm{x})(1-\mathrm{x})(1-\mathrm{y}) + (1-\mathrm{x})(0)(\mathrm{y}) + (1-\mathrm{x})(0)(1-\mathrm{y})$$

$$= -\mathrm{xyc} + \mathrm{x}(1-\mathrm{c})(1-\mathrm{y}) + 0 + 0$$

$$= \mathrm{x}(1-\mathrm{c})(1-\mathrm{y}) - \mathrm{xyc}$$

$$\mathrm{U_{p1}} = \mathrm{x}(1-\mathrm{y}-\mathrm{c}) \quad (3.2)$$

As it is mentioned above, we are using the non-cooperative approach of game theory. Therefore, every one would like to maximise its utility. Now we find the best response of player P_2 against the strategy of player P_1. From equation (3.2), if we assume that $y < 1 - c$ then the result of $(1 – c - y)$ is always positive value, and the utility of player P_1 can be maximised by setting the value of x as high as possible (maximum 1.). Conversely, if we find $(y > 1 – c)$ then the utility of player P_1 can be maximized only if we set the value of $x = 0$. Both of these cases take us back to the second chapter, where we discussed the pure strategy Nash Equilibria (T, Q) and (Q, T). To find the mixed strategy Nash Equilibria we differentiate the above equation (3.2), and get the best response of the Player P_2.

$$\frac{\partial \mathrm{U_{p1}}}{\partial \mathrm{x}} = \frac{\partial}{\partial x}\big(\mathrm{x}(1-\mathrm{y}-\mathrm{c})\big)$$

$$= 1 - y - c$$

$$1 - y - c = 0$$

$$y = 1 - c \tag{3.3}$$

Similarly, for the utility equation of player P_2

$$U_{p2} = y(1 - x - c) \tag{3.4}$$

Same as we differentiated above, we differentiate here utility equation of player P_2 and find the mixed strategy Nash equilibrium

$$\frac{\partial U_{p2}}{\partial y} = \frac{\partial}{\partial y}\big(y(1 - x - c)\big)$$

$$= 1 - x - c$$

$$1 - x - c = 0$$

$$x = 1 - c \tag{3.5}$$

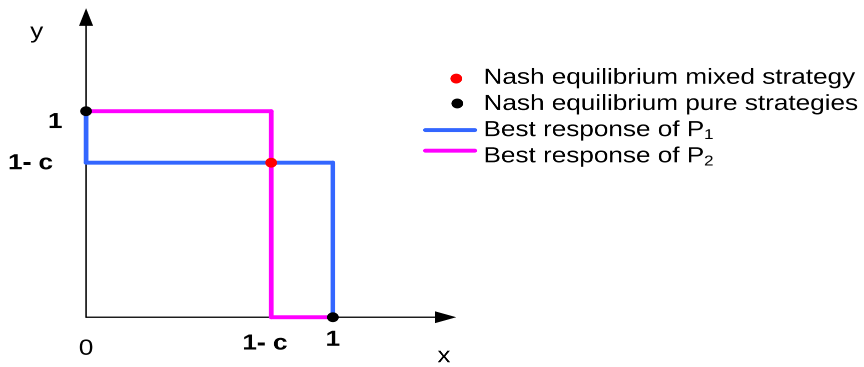

Figure 8. Best Response function

3.3.1.2 Mixed strategy joint Nash Equilibria

In this subsection, we find the mixed strategy joint Nash Equilibria by adding the utilities of both players P_1 and P_2.

$$U_{p1} = x(1 - y - c)$$

$$U_{p2} = y(1 - x - c)$$

$$U = U_{p1} + U_{p2}$$

$$= x(1 - y - c) + y(1 - x - c)$$

$$= \mathrm{x} - \mathrm{xy} - \mathrm{xc} + \mathrm{y} - \mathrm{xy} - \mathrm{yc}$$

$$\mathrm{U} = \mathrm{x} + \mathrm{y} - 2\mathrm{xy} - \mathrm{xc} - \mathrm{yc}$$

$$\frac{\partial \mathrm{U}}{\partial \mathrm{x}} = \frac{\partial}{\partial x}(\mathrm{x} + \mathrm{y} - 2\mathrm{xy} - \mathrm{xc} - \mathrm{yc})$$

$$= 1 - 2\mathrm{y} - \mathrm{c} + 0 - 0$$

$$= 1 - 2\mathrm{y} - \mathrm{c}$$

$$1 - 2\mathrm{y} - \mathrm{c} = 0$$

$$\mathrm{y} = \frac{1-\mathrm{c}}{2} \quad (3.6)$$

Now, it is possible to compare the utility of any player by considering the mixed strategy individual Nash Equilibria and mixed strategy joint Nash Equilibria. We know, in equation (3.2), if y= 1-c then the utility of player P_1 does not depend on x, it becomes zero. But if we use the mixed strategy joint Nash Equilibrium as $\mathrm{y} = \frac{1-\mathrm{c}}{2}$, which was $(y = 1 - c)$ then the utility of player still depends on the value of x, as $\mathrm{U}_{\mathrm{p}1} = x\left(\frac{1}{2} + \frac{c}{2} - c\right)$

Similarly, using mixed strategy joint Nash Equilibria for the opponent player of P_1 is as

$$\mathrm{x} = \frac{1-\mathrm{c}}{2} \quad (3.7)$$

$$\mathrm{U}_{\mathrm{p}2} = y\left(\frac{1}{2} + \frac{c}{2} - c\right) \quad (3.8)$$

3.3.1.3 Utility equations for two users on two channels

Now we step up and assume that we have two channels available for the transmission and both players compete for both of the channels to maximise their utilities. Let's say ch_1 and ch_2 are two available channels, and P_1 and P_2 are the two greedy players, who want to access them, basically in this scenario we assume that x_1 and x_2 are the transmission probabilities of player P_1 on channel ch_1 and ch_2 respectively, similarly y_1 and y_2 are the transmission probabilities of player P_2 on channels ch_1 and ch_2. Where$(1 - \mathrm{x}_1 - \mathrm{x}_2)$and $(1 - \mathrm{y}_1 - \mathrm{y}_2)$ are the probabilities of being quiet of player P_1 and P_2.

$P_1 \backslash P_2$	(Ch_1) y_1	(Ch_2) y_2	Quiet $(1 - y_1 - y_2)$
(Ch_1) x_1	$(-c, -c)$	$(1-c, 1-c)$	$(1-c, 0)$
(Ch_2) x_2	$(1-c, 1-c)$	$(-c, -c)$	$(1-c, 0)$
Quiet $(1 - x_1 - x_2)$	$(0, 1-c)$	$(0, 1-c)$	$(0, 0)$

Table 3.2. Two channels for two players.

$$U_{p1} = x_1(-c)y_1 + x_1(1 - c)y_2 + x_1(1 - c)(1 - y_1 - y_1)$$

$$+x_2(1 - c)y_1 + x_2(1 - c)y_2 + x_2(1 - c)(1 - y_1 - y_1)$$

$$= x_1[-cy_1 + y_2 + y_2c + 1 - y_1 - y_2 - c - y_1c - y_2c]$$

$$+ x_2[y_2 - cy_1 - y_2c + 1 - y_1 - y_2 - c - y_1c - y_2c)]$$

$$U_{p1} = x_1(1 - y_1 - c) + x_2(1 - y_2 - c) \quad (3.9)$$

From above equation, we can calculate the utility for any player from both channels, it just adds up the utilities and return to the player. We find 4 pure strategy Nash Equilibria and to calculate the mixed strategy Nash Equilibria, we assume that the player P_1 and P_2 has the same probabilities of transmission on both channels, it can be written as x_1=x_2=x and y_1=y_2=y, so the utility equation for player P_1 is:

$$U_{p1} = x(1 - y - c) + x(1 - y - c)$$

$$U_{p1} = 2x - 2xy - 2xc$$

$$\frac{\partial U_{p1}}{\partial x} = \frac{\partial}{\partial x}(2x - 2xy - 2xc)$$

$$= 2 - 2y - 2c$$

$$= 2(1 - y - c)$$

$$2(1 - y - c) = 0$$

$$y = \frac{1-c}{2} \quad (3.10)$$

similarly,

$$x = \frac{1-c}{2}$$

3.3.1.4 Utility equations for three users on a common channel

It is assumed that x, y and z are the transmission probabilities for player P_1, P_2, and P_3 respectively. Therefore, from equation 2.1, we can calculate the utilities of each player, such as P_1 has a utility as

$$U_{p1} = x(-c)(y)(z) + x(-c)(y)(1-z) + x(-c)(1-y)(z) + x(1-c)(1-y)(1-z)$$

$$U_{p1} = x[-yzc - yc(1-z) + (1-c)(1-c-y-z+yz)]$$

$$U_{p1} = x[-yzc - yc + ycz + 1 - y - z + yz - c + zc + yc - yzc]$$

$$U_{p1} = x(1 - y - z + yz - c) \quad (3.11)$$

Similarly,

$$U_{p2} = y(1 - x - z + xz - c) \quad (3.12)$$

$$U_{p3} = z(1 - x - y + xy - c) \quad (3.13)$$

Example 3.1

In the following part, we take a very simple example and discuss that how Nash equilibrium works with the reward table.

- In table (3.3), it is shown that two players with two actions (0 or 1), their overall utility becomes highest if both players chose action 0 for their data transmission. But if any of the players transmit very aggressively using action 1 and mean time the other player transmit with action 0 (less aggressively), the aggressive player will get higher utility but the overall collective sum of utility will become lower than first case, and vice versa for the other player. In fourth option, if both of the players transmit their data aggressively by choosing action 1, they will experience collision and the overall utility will become very low (almost zero).

$P_1 \backslash P_2$	0	1
0	(5, 5)	(3, 6)
1	(6, 3)	(0, 0)

Table 3.3: Reward table[5]

- In pure strategy Nash equilibrium, either of the players will get a utility of 1 if its opponent keeps quiet. For instance, if both players transmit with the actions of 0 at same time or with the actions of 1 at the same time they will get a utility of 0. Therefore, we can say that we have two pure strategy Nash Equilibria (0, 1) and (1, 0) in two players multiple access game.

$P_1 \backslash P_2$	0	1
0	(0, 0)	(0, 1)
1	(1, 0)	(0, 0)

Table 3.4: Pure Strategy Nash Equilibrium[5]

- We know that in mixed strategy games, players have some probabilities for some actions, to calculate the probability of transmission; we assume that x is the probability of transmission of player P_1 and y is the probability of transmission of player P_2. Therefore, according to equation 2.1 and table 3.2, we have

$$\mathrm{U}_{\mathrm{p}1} = (1-x) * (1-y) * 5 + x * (1-y) * 6 + (1-x) * y * 3$$

$$= (1-x-y+xy) * 5 + (x-xy) * 6 + (y-xy) * 3$$

$$= 5 - 5x - 5y + 5xy + 6x - 6xy + 3y - 3xy$$

$$\mathrm{U}_{\mathrm{p}1} = 5 + x - 2y - 4xy$$

Differentiating above equation with respect to x.

$$\frac{\partial \mathrm{U}_{\mathrm{p}1}}{\partial x} = 0 + 1 - 0 - 4y$$

Assume that

$$1 - 4y = 0$$

$$y = \frac{1}{4} = 0.25$$

Similarly, we can calculate the probability of transmission x for player P_1, and we get

x= 0.25

Now we have information about the probability of transmission of both players, therefore, the probability of being quiet for player P_1 and P_2 are as:

$1 - x = 0.25$

And $1 - y = 0.75$

$P_1 \backslash P_2$	0 (0.75)	1 (0.25)
0 (0.75)	9/16	3/16
1 (0.25)	3/16	1/16

Table 3.5: Mixed Strategy Nash Equilibrium [5]

Therefore, using the mixed strategy Nash equilibrium we can calculate the utility for each user, with the help of table 3.2 and 3.4, we have

$$U_{p1} = (0.75 * 0.75 * 5) + (0.75 * 0.25 * 3) + (0.25 * 0.75 * 6)$$

$$= 2.8125 + 0.5625 + 1.125$$

$$U_{p1} = 4.5 \quad (3.14)$$

Therefore, with mixed strategy Nash equilibrium, we have a utility of 4.5.

4 INVESTIGATION OF MULTIPLE ACCESS GAME WITH CORRELATED EQUILIBRIUM

4.1 Introduction

The characteristics of the opportunistic based radio spectrum access vary with respect to the time, frequency and physical locations [4] . Artificial scarceness of the spectrum is not only due to the increasing use of spectrum but also due to the limited access of spectrum to newly developed technologies. The shortage of the spectrum can be alleviated by sharing and accessing the spectrum opportunistically when it is not in use of its owners [4]. A sensing based approach can be used to access the spectrum opportunistically, for this purpose spectrum-agile devices are attached which needs very low and simple infrastructure requirements [19]. Asynchronous Distributed Pricing (ADP) to control the distributed interference when multiple distributed users access the common channel[20].

We recall the basic idea behind the conception of correlated equilibrium; it was introduced by Aumann in about 70s. As it is stated earlier in section (2.2.10), the fundamentals of this phenomenon are as, that private instruction helps the player in choosing or making his decision for his action [18]. This approach is very similar to the Nash equilibrium but addition is private message or signal. All privately received messages take part in the generation of all the Correlated Equilibria. The Nash equilibrium and correlated equilibrium divides their ways as, if the privately received messages or signals for a player does not depend on the signals received by any other player, it means that it produces the Nash equilibrium of game which can be under the pure or mixed strategies. But there can be a possibility that the privately received messages are correlated with each other, in that case the Equilibria achieved from those strategies selected by the correlated messages can be different [18].

In section (4.2), we model our problem and describe the interference matrix, channel availability matrix, and a strategy space matrix which hold the information about all the possible actions which we can use in our game. Correlated Equilibrium is defined and implemented in section (4.3) with the help of a very simple example, where we compare the utility achieved using the Correlated Equilibrium and Nash Equilibrium.

4.2 Opportunistic Spectrum Access using Game theory

In our network model, we assume that we have N available channels from Fixed Satellite Service

(FSS) and each of these hold the unit bandwidth. There are M primary (FSS users) users who are sharing all these channels and K secondary users (RAN_1 and RAN_2), trying to access the channels opportunistically when they get free. Briefly, to measure the interference for neighbouring secondary users with each other, we build a matrix of interference called L, and the dimensions of this matrix should be K x K [5].

$$L_{i,j} = \begin{cases} 1, & \text{if } i \text{ and } j \text{ interfere with each other} \\ 0, & \text{otherwise} \end{cases} \tag{4.1}$$

This newly born matrix totally depends on the positions of users. In our scenario, both of the cells overlap with each other, therefore, there is definite interference. Another important parameter of network model is to find the channel availability matrix. The dimension of the channel availability matrix A(t) can be K x N, we already know that K represents the total number of secondary users and N, number of available channels. Different rates can also be defined for all users on any specific channel with which they can transmit their data. So the channel availability matrix [5] can be defined as:

$$A_{in}(t) = \begin{cases} 1 & \text{if channel n is available for any secondary user } i \text{ at a perticulary time t:} \\ 0 & \text{Otherwise} \end{cases} \tag{4.2}$$

It is known, that the matrix A(t), channel availability always changes over time to time. The results for the above mentioned matrix are based on secondary user's sensing task and on the traffic from the primary users. This whole theory is implemented in the following example.

Generic representation of secondary users can be, lets suppose we have K secondary users I = 1, 2, 3,……..K. Different data rates can be selected on every available channel by secondary users, a set of data rates can be defined as $[0, v_1, v_2, v_3, \dots\dots\dots]$. All the secondary users can have their own strategy spaces on the available channels, here we define the secondary user i's strategy space in this form.

$$\Omega_i = \prod_{n=1}^{N} \gamma^{A_{i,n}} \tag{4.3}$$

Secondary user's action can be represented as $r_i^n = v_i$, which shows that secondary user i utilises the available channel n with a rate of v_i. the strategy profile for any channel n can be defined as

$\mathrm{r}^n = \mathrm{r}_1^n, \mathrm{r}_2^n, \mathrm{r}_3^n, \mathrm{r}_4^n, \dots \dots \dots \dots \dots \mathrm{r}_i^n$, the strategies of opponent users for user *i* can be written as r_{-i}^{n}, similarly we define the actions of user *i* for all available channels as $\mathrm{r}_i = \mathrm{r}_i^1, \mathrm{r}_i^2, \mathrm{r}_i^3, \mathrm{r}_i^4, \dots \dots \dots \dots \dots \mathrm{r}_i^N$. Now we measure the utility for secondary user *i* using different strategies over different channels using the utility function U_i.

$$\mathrm{U}_i = \sum_{n=1}^{N} \mathrm{A}_{i,n} \mathrm{R}_i(\mathrm{r}_i^n, \mathrm{r}_{-i}^n) \tag{4.4}$$

Where $R_i(r_i^n, r_{-i}^n)$ returns value after resource competition when secondary user *i* competes with all other secondary users who are using the particular channel n at time t. another parameter mentioned in above equation is the r_i^n which show actually that what rate was chosen by secondary user *i* over channel n, higher rate produces the high utility, the value of R_i can be calculated from the following equation [9].

$$\mathrm{R}_i(r_i^n, r_{-i}^n) = \begin{cases} \frac{r_i^n S^n}{\sum_i r_i^{n,}}, & \mathrm{G} \leq \mathrm{Go} \\ 0, & \text{otherwise} \end{cases} \tag{4.5}$$

In equation (3.5), we have number of important parameters, such as $\mathrm{r}_\mathrm{i}^\mathrm{n}$ (mentioned above), $\sum_s \mathrm{r}_\mathrm{s}^{\mathrm{n},}$ shows the data rate on channel n from all users including secondary user *i* and its opponents. G_O is the maximum bandwidth of channel n, if load on available channel n increases from G_O then there can be a collision which will cause unacceptable delay. S^n in [9] described here as

$$S^n = \frac{G^n\,[1+G^n+\tau G^n\,(1+G^n+\tau G^n/2\,)]e^{-G^n(1+2\tau)}}{G^n\,(1+2\tau)-\left(1-e^{-\tau G^n}\right)+(1+\tau G^n)e^{-G^n(1+\tau)}} \tag{4.6}$$

Here $G^n = \sum_i r_i^{n,}$ show again the sum of data rate on channel n from secondary user *i* and its opponents, where τ is described as the packet delay on the transmission time. When load on the network spikes up then more collisions occur, then the packet delay for each user increases instantly and quality of service decreases.

Up to this stage, we are able to calculate the utility of any secondary users, who is accessing the channel opportunistically. In the following sections we will discuss different methods of maximising the secondary user's utility, and will find the optimum solution.

4.3 Correlated Equilibrium:

Correlated equilibrium, as compared to the Nash equilibrium, is more general form to represent the utility. In this strategy, with respect to any particular distribution, a randomly picked strategy is chosen which is to the player's best attention to obey the rules with this strategy [7]. This game consists of different parameters such as total number of secondary users, their strategy spaces and the utilities. If we consider our game as $G^{'}$ then correlated strategy can be defined as "*A probability distribution p is a correlated strategy of game $G^{'}$ if $\forall i \in K$, $\forall r_i^{'} \in \Omega_i$, $\forall r_{-i} \in \Omega_{-i}$.*" [5].

Mathematically it can be represented as

$$\sum_{r_{-i} \in \Omega_{-i}} p(r_{-i}|r_i)[U_i(r_i^{'}, r_{-i}) - U_i(r_i, r_{-i})] \leq 0 \quad \text{for all } r_i^{'} \in \Omega_s . \tag{4.7}$$

In above equation, it is clear that if player *i* choose the action r_i then choosing action $r_i^{'}$ instead of action r_i cannot give higher utility [7]. Now we go back to our problem in table (3.3), and solve it with the help of correlated equilibrium. It is assumed that, there are two secondary players and two different actions, a right action at right time can produce a higher utility. It is known that, there are three Nash Equilibria in Multiple access scenario for two players, two of them are pure strategy Nash Equilibria (1, 0), (0, 1) and one is mixed strategy Nash Equilibria. In pure strategy Nash Equilibria only one of the players can transmit at particular time but in mixed strategy Nash Equilibria, both of the players can transmit with some probabilities and get the utilities which they deserve according to their actions. With respect to the equation (4.7), it is possible to set up some conditional probabilities to all possible of the actions.

$P(P_1 = 0 \mid P_2 = 0) = 0.60$

$P(P_2 = 0 \mid P_1 = 0) = 0.60$

$P(P_1 = 0 \mid P_2 = 1) = 0.20$

$P(P_1 = 1 \mid P_2 = 0) = 0.20$

$P(P_1 = 1 \mid P_2 = 1) = 0.00$

$P_1 \backslash P_2$	Action 0	Action 1
Action 0	0.60	0.20
Action 1	0.20	0.00

Table 4.1: Correlated Equilibrium [5]

Utility calculation:

According to definition of Correlated Equilibrium, now it is possible to calculate the utility of any player with the help of equation (2.1), table (4.1) and (3.3).

$$U_{p1} = (5 * 0.60) + (3 * 0.20) + (6 * 0.20)$$

$$U_{p1} = 4.8 \quad (4.8)$$

Now by comparing both of the utilities taken through mixed strategy Nash equilibrium (equation (3.14)) and Correlated Equilibrium (equation (4.8)), we can conclude that utility using the Correlated Equilibrium is 6.67% larger than the mixed strategy Nash Equilibrium.

4.4 No-Regret Learning Algorithm

No regret strategy, based on regret-matching algorithm [10]. This algorithm actually shows no-regret and play probabilities are proportional to the regrets for having not played the other possible actions. It is assumed here a strategy space Ω_i of having distinct action $r_i \neq r_i'$, then the regret of player *i* for not playing r_i' at time T.

$$\mathbb{R}_i^T(r_i, r_i') := \max\{Q_i^T(r_i, r_i'), 0\} \quad (4.9)$$

Where

$$Q_i^T(r_i, r_i') = \frac{1}{T} \sum_{t \le T} \left(U_i^t(r_i, r_i') - U_i^t(r_i, r_{-i}) \right) \quad (4.10)$$

Here $Q_i^T(r_i, r_i')$ can be represented as the average utility which player *i* will achieve. If player *i* plays the action r_i' all times instead of selecting r_i then the value of $\mathbb{R}_i^T(r_i, r_i')$ will be assumed as the average regret. In the following algorithm the probability of choosing action r_i is a linear function of the regret [5].

Algorithm 4.1

Initialize arbitrarily probability for taking action of player i,$p_i^1(r_i), \forall_i \in K$
For t = 1,2,3,…..

1. Find $Q_i^T(r_i, r_i')$ as in (4.10)
2. Find average regret $\mathbb{R}_i^T(r_i, r_i')$ as in (4.9)
3. Let $r_i \in \Omega_i$ be the strategy last chosen by player i,
   i.e. $r_i^t = r_i$. then the probability distribution of the actions for the next period, p_i^t is defined as
   $p_i^t(r_i') = \frac{1}{\mu} \mathbb{R}_i^T(r_i, r_i') \quad \forall r_i' \neq r_i,$
   $p_i^t(r_i) = 1 - \sum_{r_i' \neq r_i} p_i^t(r_i'),$
   Where μ is a certain constant that is sufficiently large.

Now for each period of time T we define the relative frequency of players joint action r played till T period of time as follows

$$z_T(r) = \frac{1}{T} \#\{t \leq T : r_t = r\} \qquad (4.11)$$

Where $\#(\cdot)$ denotes the number of times the event inside the bracket happens and r_t is all users action at time t. it is show in the following plot that z_T converges to one of the pure strategy Nash Equilibrium if each player select its action with respect to the above algorithm.

The following simulations are designed to investigate the behaviour of an adaptive learning algorithm with the passage of number of experiments. Simulations represent the visual result of algorithm (4.1) and equation (4.11).

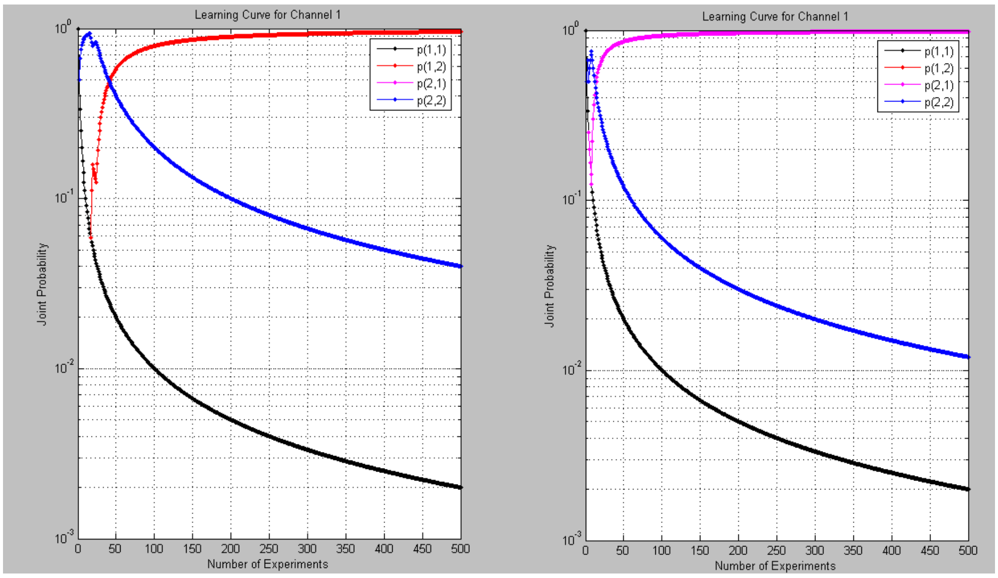

Figure 9. Adaptive Convergence to Nash Equilibrium

Figure 9. shows the joint probability distribution behaviour of two operator on a common channel Four possible combinations of attempts are designed for both players, the actions we have selected are (0.5 and 1), represented as (1 and 2) respectively. From the above figure , it is clearly shown, that given any arbitrary point of the game that is played repetitively, the adaptive algorithm is capable to converge to one of the pure strategy Nash Equilibrium in the game. As we know that the pure strategy Nash Equilibria produces the optimum utility.

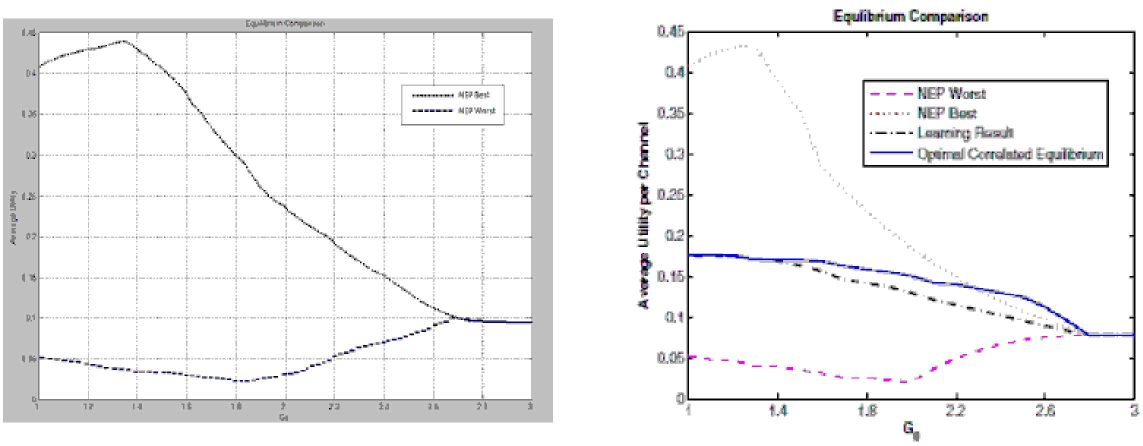

Figure 10. Utility function Vs Go Three users. UniS (left) & Ref [5] (Right)

In both figures (10), the results of different Equilibria as a function of Go is investigated for three players. Using the action space of $[0.05, 0.06, 0.07, \ldots \ldots \ldots, 1.5]$ and delay factor 0.005 ($\tau = 0.005$), we find the gain achieved by aggressive player in the Nash Equilibrium Point (NEP) and also gain achieved by the less aggressive player or victim of the aggressive player in the NEP. When the value of Go is large, the aggressive user face a small penalty for his greedy behaviour. Just before the value 2.8 of Go, the aggressive player have very good performance which is called the NEP Best but the opponent player who is very cooperative with aggressive player, transmits less aggressively and due to that reason he gets the worst NEP. In graph, we show that no player can increase his own utility by changing his strategy action without affecting the utility of other players in the game, which in fact justify the definition of Nash Equilibrium.

5 CONCLUSION AND FUTURE WORK

5.1 Conclusion

Today, wireless communication provides very advanced services which play very important role in our daily life. Due to the technological advancements, very sophisticated and programmable devices are available in market for normal users, operators or service providers. And all these things are leading to the scarceness of the radio spectrum. New service operators can deploy their infrastructure very easily and stand in market in front of large operators. In this project, our main objective was to analyse the behaviour of rational players in the wireless communication and we relied of the non-cooperative game theory.

Different scenarios of wireless communications are investigated under the influence of game theory. Based on multiple access game, pure strategy Nash Equilibrium produces the optimum utility of 1. In conflicting situations, players use different action with variable probabilities which leads to the mixed strategy. Mixed strategy Nash Equilibrium is investigated with the help of best response function. We optimize the individual and joint Nash Equilibria using the mixed strategy and find that the mixed strategy joint Nash Equilibria gives better utility having same probabilities of transmission as for mixed strategy individual Nash Equilibria. In competitive wireless environment we evaluated the interference and channel availability matrices. And calculate the utilities by adopting different possible rates. Correlated Equilibrium produces 6.67% more utility as compare to the mixed strategy Nash Equilibrium. Different conditions are obeyed to achieve the utility above mentioned. Adaptive learning algorithm is used as well to achieve the convergence of joint probability distribution for multiple players. In fact, it shows that given any arbitrary strategy point of the game that is played over and over again, the adaptive learning algorithm can converge to one of the pure strategy Nash Equilibrium of the game. Linear programming solution is used as to optimize the utility of all players on the available channels.

Sharp rise in new wireless technologies has increased the competition for the access of radio spectrum. Therefore, to manage the spectrum more efficiently, it is very interesting to investigate in the following areas.

5.2 Future Work

Future work based on this project can be distributed in the following manner:

5.2.1 Modified No-Regret Algorithm

From our investigation about the No-regret learning algorithm, we show that, it converges to one the pure strategy Nash Equilibrium of the game not to the Correlated Equilibrium. Therefore, it needs some modifications which can lead to the Correlated Equilibrium.

5.2.2 Cost

Our main objective in this project was the utility of player where cost plays very important role. In our examples (used as game theory in wireless communications) the cost was assumed as just the energy utilization of devices. But it can be different, we may have power plugged devices or those devices whose cost increases if their battery is very low powered and cost decreases if their battery is fully charged. Therefore, cost can play very important role in opportunistic access of the spectrum

5.2.3 Cooperative players

In our project, we assumed all of our players as rational. They tried to maximize their own individual utilities and did not cooperate with each other, the objective of our this approach was to reduce the signalling overhead. It is also possible to do some work on cooperative games where bargaining and coalition issues can be discussed.

REFERENCES

[1] Mark McHenry. "Spectrum white space measurements", *presented to New America Foundation BroadBand Forum,* June 2003.
[online]
http://www.newamerica.net/Download_Docs/pdfs/DocFile_185_1.pdf [accessed 27/05/2009]

[2] M. Felegyhazi, Jean-Pierre Hubaux, "Game Theory in Wireless Networks: A Tutorial" *EPEL Technical report: Switzerland,* Jun. 28, 2006.

[3] J. Nash, "Equilibrium points in n-person games", *proceedings of the National Academy of Sciences,* vol. 36, pp.48-49, 1950.

[4] X. Liu and W. Wang, "On the characteristics of spectrum-agile communication networks", *the first IEEE International Sysmposium on New Frontiers in Dynamic Spectrum Access Networks,* 2005 (DySPAN 2005), pp. 214-223, Baltimore, MD, November 2005.

[5] Z. Han, C. Pandana and K. J. Ray Liu, "Distributive Opportunistic Spectrum Access for Cognitive Radio using Correlated Equilibrium and No-regret Learning, presented" *in proceeding of IEEE WCNC,* 11-15 March 2007. Digital Object Identifier: 10.1109.WCNC.2007.8 .

[6] S. V. Maric and O. Moreno, "Using Costas Arrays to Construct frequency Hop Pattern for OFDM Wireless Systems", *Conference on Information Science and Systems, CISS 2006, Princeton NJ USA* .

[7] [online] http://en.wikipedia.org/wiki/Correlated_equilibrium. [accessed 19/07/2009]

[8] W. Chenwei, Z. Xin, Y. Dacheng, "Evaluation of Welch-Costas Frequency Hopping Pattern for OFDM Cellular System", *the 18th Annual IEEE International Symposium on Personal, Indoor, and Mobile Radio Communications 2007 (PIMRC'07).*

[9] D. P. Bertsekas, R. G. Gallager, "Data Networks", *2nd edition, Prentice hall, Inc.* 1992.

[10] S. Hart, A. Mas-Colell, "A simple adaptive procedure leading to Correlated Equilibrium", *Econometrica*, vol. 68, no. 5, pp. 1127 – 1150, September 2000.

[11] Xiao, Schroff, Chong, “A utility based power control scheme in wireless cellular systems”, *IEEE/ACM Trans. On Networking,* 11(10):210-221, March 2003.

[12] P. Marbach, R. Berry. “ Downlink resource allocation and pricing for wireless networks”, *In proceeding of the IEEE Conference on Computer Communications,* New York, USA, 23-27 June 2002.

[13] C. U. Saraydar, B. Mandayam, J. Goodman, “Efficient power control via pricing in wireless data networks”, *IEEE transitions on Communications,* 50(2) : 291 – 303, 2002.

[14] Roger B. Myerson, “Game Theory Analysis of Conflict”, *First Harvard University Press edition, 1997,* pp 2-22, ISBN 0-674-34115-5 (pbk).

[15] Siegel, Siegel, and Andrews, “Choice, Strategy, and Utility”, *McGraw-Hill, Inc 1964,* pp 3-11 and pp 24-35, 57355.

[16] Z. Han, K. J. Ray Liu, “Resource Allocation for Wireless Networks, Basics, Techniques, and Applications”, *Cambridge University Press 2008,* pp 203-224, ISBN 978 0 521 87385 7. Hardback.

[17] S. Haykin, “Cognitive Radio Brain Empowered Wireless Communication”, *IEEE, Journal on Selected Areas in Communication,* vol. 23, no.2, pp.201-220, Feb 2005.

[18] R. J. Aumann, “subjectivity and Correlation in Randomized Strategy”, *A Journal of Mathematical Economics,* vol. 1, no.1, pp 67-96, 1974.

[19] X.Liu, S. Shankar, “Sensing-based Opportunistic channel access”, *ACM J. Mobile Networks 11*, 577-591 (2006)

[20] J. Huang, R. A. Berry, M. L. Honig, “Spectrum Sharing with Distributed Interference Compensation”, *in DySPAN(2005),* pp 88-93.

[21] [Online] http://en.wikipedia.org/wiki/Auction_theory, http://en.wikipedia.org/wiki/Price. [Accessed on 12/08/2009].

[22] C. U. Saraydar, N. B. Mandayam, D. J. Goodman, "Efficient Power Control via Pricing in Wireless Data Networks", *in proceeding of IEEE WCNC 1999.*

APPENDIX 1

Code for Simulation:

```
clear all

clc

N = 1;

K = 2;

Tau = .1;              %Delay parameter

Gamma = [0.5 1];  % Possible actions

Go = 2.8;              % Available Bandwidth

A = ones(K,N);

DD = cell(K,1);

Count = cell(K,1);


for j = 1:K

    len = length(find(A(j,:) == 1));

    V{j} = repmat(Gamma,1,len);

    C{j} = nchoosek(V{j},N);

%    C{j} = combnk(V{j},N);

    Omegai{j} = unique(C{j},'rows');          %% Construction of Strategy space

    [nrows, ncols] = size(Omegai{j});

    pi_t{j} = repmat(1/nrows, 1, nrows);      %% Initial arbitrary probabilities

    DD{j} = zeros(nrows,nrows);

    Count{j} = zeros(nrows,nrows);

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%


%% this part of program calculates the value of MU, which is used in
```

```
%% algorithm during the calculation of probability of choosing next possible action.
Omega = Omegai{1};
for j = 2:K
   Omega = [Omega;Omegai{j}];
end
Combi = nchoosek(Omega,K);
Actions = unique(Combi,'rows');
[nrows, ncols] = size(Actions);
for i = 1:nrows
    actions = Actions(i,:)';
   for k = 1:K
       U(k) = Utility(actions, Tau, N, K, A, k, Go);
   end
   maxU(i) = max(U);
end
Margin = 2;  % How many times Mu is lager than the minimum requirement
Mu = Margin * 2 * ((length(Gamma)^N)-1) * max(maxU);
%Mu = 1.0;
[nrows, ncols] = size(Omegai{1});
%temp = randsrc(1,1,[1:nrows;pi_t{j}],543677);
iterations = 500;
%% Experiments start from here using different actions
for T = 1:iterations
    for j = 1:K  %%K denotes the number of players participate in game.
        actions_index{j} = randsrc(1,1,[1:nrows;pi_t{j}]);  %% Random Index selection of action
        actions(j,:) = Omegai{j}(actions_index{j},:);Appropriate action, according to the Index.
        ri(j,:) = actions(j,:);
```

```
    %%Index selection of second possible action
    actions_b_indices{j} = find( (actions_index{j} - (1:nrows)) ~= 0);
    len = length(actions_b_indices{j});
    for k = 1:len
        ri_b{j}(k,:) = Omegai{j}(actions_b_indices{j}(k),:);%%Appropriate second action.
    end
    actions_hist{j}(T,:) = actions(j,:);
end
for j = 1:K
    actions_b = actions;
    U(j,T) = Utility(actions, Tau, N, K, A, j, Go);%% Utility calculation using the first actions of
%%all players in game.
    sum_actions_b = 0;
    for k = 1:len
        actions_b(j,:) = ri_b{j}(k,:);
        U_b(j,T) = Utility(actions_b, Tau, N, K, A, j, Go);%%Utility, using second action.
        DD{j}(actions_b_indices{j}(k), actions_index{j}) = DD{j}(actions_b_indices{j}(k), ac-
tions_index{j}) + (U_b(j,T) - U(j,T));
        Count{j}(actions_b_indices{j}(k), actions_index{j}) = Count{j}(actions_b_indices{j}(k),
actions_index{j}) + 1;
        D{j}(T) = DD{j}(actions_b_indices{j}(k), actions_index{j}) /
Count{j}(actions_b_indices{j}(k), actions_index{j});
        R{j}(T) = max(D{j}(T), 0);  %% Regret selection
        pi_tp1_b{j} = R{j}(T) / Mu; %% Probability calculation of choosing action-2.
        pi_t{j}(1,actions_b_indices{j}(k)) = pi_tp1_b{j};
        sum_actions_b =sum_actions_b + pi_tp1_b{j};
    end
    pi_tp1{j} = 1 - sum_actions_b; %%Probability of selecting action-1.
```

```
        pi_t{j}(1,actions_index{j}) = pi_tp1{j};
    end
%%%%%%% Joint probability calculations.
    for i = 1:N
        p11(i,T) = length(find( (actions_hist{1}(:,i) == 0.5) & (actions_hist{2}(:,i) == 0.5) )) / T;
        p12(i,T) = length(find( (actions_hist{1}(:,i) == 0.5) & (actions_hist{2}(:,i) == 1) )) / T;
        p21(i,T) = length(find( (actions_hist{1}(:,i) == 1) & (actions_hist{2}(:,i) == 0.5) )) / T;
        p22(i,T) = length(find( (actions_hist{1}(:,i) == 1) & (actions_hist{2}(:,i) == 1) )) / T;
    end
end
%% Plotting.
for i = 1:N
    figure;
    subplot(121);
    semilogy(1:iterations, p11(1,:), 'k.-');
    hold on;
    grid on;
    semilogy(1:iterations, p12(1,:), 'r.-');
    semilogy(1:iterations, p21(1,:), 'm.-');
    semilogy(1:iterations, p22(1,:), 'b.-');
    xlabel('Number of Experiments');
    ylabel('Joint Probability');
    title(['Learning Curve for Channel ', num2str(i)])
    legend('p(1,1)','p(1,2)','p(2,1)','p(2,2)')
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%%Utility Function.

function Ui = Utility(actions, tau, N, K, A, j, Go)

% actions should by 1 * N matrix

for i = 1:N

    Gn = sum(actions(:, i));

    Sn_nu = Gn*(1 + Gn + tau*Gn*(1 + Gn + (tau*Gn)/2))*exp(-1*Gn*(1+2*tau));

    Sn_de = Gn*(1+2*tau) - (1-exp(-1*tau*Gn))+(1+tau*Gn)*exp(-1*Gn*(1+tau));

    Sn = Sn_nu/Sn_de;

    if Gn <= Go

        Ri(j, i) = actions(j, i) * Sn / Gn;

    else

        Ri(j, i) = 0;

    end

end

Ui = sum( (Ri(j,:) .* A(j,:)) );


if actions == [0.5;0.5]

    if j == 1

%        Ui = 5;

        Ui = 0;

    elseif j == 2

%        Ui = 5;

        Ui = 0;

    end

elseif actions == [0.5;1]

    if j == 1

%        Ui = 3;
```

```
        Ui = 0;
    elseif j == 2
%       Ui = 6;
        Ui = 0.9;
    end
elseif actions == [1;0.5]
    if j == 1
%       Ui = 6;
        Ui = 0.9;
    elseif j == 2
%       Ui = 3;
        Ui = 0;
    end
elseif actions == [1;1]
    if j == 1
%      Ui = 0;
        Ui = -0.1;
    elseif j == 2
%       Ui = 0;
        Ui = -0.1;
    end
end
```