# Performance of Multicell Joint Processing Cellular Uplink in the Presence of Relay Nodes

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**Abstract:** In this paper we compare the uplink performance of various deployment scenarios of a linear cellular system where the received signals of multiple cells are processed jointly at a central processor. A mathematical model is developed and the performance is compared using the information theoretic sum rate and user rate distribution for the system. A benchmark system deployment of a linear cellular array is changed in the following manners: the density of the base stations is decreased, relay stations replace alternate base stations and in another scenario the alternate base stations are removed while an additional antenna is placed on each left over base station in the system. It is observed that decreasing the density of base stations reduces the sum rate and makes user rate shares relatively unfair. Replacing base stations by the relays and use of amplify and forward scheme reduces the sum capacity (to a value approximately same as for the case where alternate base stations are removed) but maintains a relatively fair user rate distribution. Replacing a base station with an additional antenna maintains the sum-rate of the system but user rate share for the users at the edges of the cell is reduced. The results suggest that the relay deployment in the multicell joint processing system does not improve the sum rate of the system rather it improves the rate share for the cell edge user making the user rate distribution fairer.

**Keywords:** Uplink, Cellular, Relay, Base Station Cooperation

## 1. Introduction

Performance improvement of wireless cellular systems is becoming increasingly more important with the emerging trend of using high-rate data services on the move. Inter-cellular interference has been a big obstacle in obtaining high spectrum efficiency for the cellular system and the performance of these systems is usually interference limited. Multicell joint processing, as originally proposed by Wyner [1] and Hanly-Whiting [2], is a promising approach to achieve higher spectrum efficiency in the cellular systems.

Wyner in [1], studies the benefits of multicell joint processing assuming an additive white Gaussian noise (AWGN) channel with no Rayleigh fading. Somekh and Shamai [3] extend Wyner's model by incorporating the narrowband single-tap Rayleigh fading. Extended models for progressively more practical considerations are studied in [4, 5]. They all show the potential improvements in spectrum efficiency when multicell joint processing is deployed.

Recently there has been an interest in the deployment of Relay nodes to improve the performance of the cellular systems and some studies were performed for the use of relay nodes in the context of multicell joint processing [6, 7] where both Amplify and Forward

[6] and Compress and Forward [7] schemes have been studied. Simeone et al [6] consider a TDMA cellular system and for keeping the model mathematically tractable they assume that the users in each cell have the same relative position with respect to the closes base station. Distance dependent path loss is modeled with a single system-wide parameter. The main finding is that the benefit of Amplify and Forward relaying is limited to the regime of low to moderate transmission rates. In [7] the authors show that the Compress and Forward scheme is capable of totally eliminating the inter-relay interference and performs better than the Amplify and Forward scheme over a wide range of parameters.

In this paper we extend the model by introducing the distance dependent path loss and allowing for any user location within each cell. The restriction of serving one user at a time in each cell is also removed. As in [4, 6, 7], we consider a linear cellular system to keep the analytical exposition simpler to follow. All the base stations are assumed to be connected to a central processor through a high capacity and delay-less back-haul. The relays are positioned at the midpoint between the two base stations. Wireless channel with narrowband single tap Rayleigh fading is considered and information theoretic sum rate and user rates are compared to estimate the performance of several deployment options.

The rest of the paper is structured as follows. The next section describes the system model followed by the section that describes how information theoretic sum-rate and user rates are calculated. Some results and insights are discussed for selected scenarios in Section 4. The work is concluded and some future work directions are identified in the last section.

## 2. System Model

We present the system model used to calculate the fundamental performance limits. A linear cellular array of $N$ cells (each indexed by $n$) is considered. The base station are deployed at equal distance on a straight line and users are also located at any point on the line. Each cell is served by a single Base Station (BS) located in the centre of the coverage extent of the cell. We assume that there are $K$ users (each indexed by $k$), in each cellular extent, which are simultaneously being served. Each user terminal and each BS has a single antenna.

We develop a general model for communication from $S$ sources to $D$ destination receivers. A signal $x$ (a complex number in base-band model) is transmitted with the constraint that $\mathbb{E}[xx^{\dagger}] \leq P$ where $P$ is the per source transmit power constraint and $\mathbb{E}$ represents the expectation of a random variable. The channel gain from each source $s$ to each destination $d$ is given by $h_{d,s}$. Let the vector $\mathbf{y}$ represent all the received signals, at the destination nodes, stacked together. By stacking all transmitted signals from the sources, we form a vector $\mathbf{x}$. Using the channel matrix $\mathbf{H}$ formed by placing all the channel gains at appropriate row-column position, we obtain the following matrix equation:

$$\mathbf{y} = \mathbf{H}\mathbf{x} = \mathbf{w} \tag{1}$$

where $\mathbf{w}$ is the Additive White Gaussian Noise (AWGN) vector for the destination nodes, $\mathbf{R}_{\mathbf{w}} \triangleq \mathbb{E}[\mathbf{w}\mathbf{w}^{\dagger}] = \sigma_0^2 \mathbf{I}_D$ and $\mathbf{I}_D$ is the identity matrix of dimensions $D \times D$.

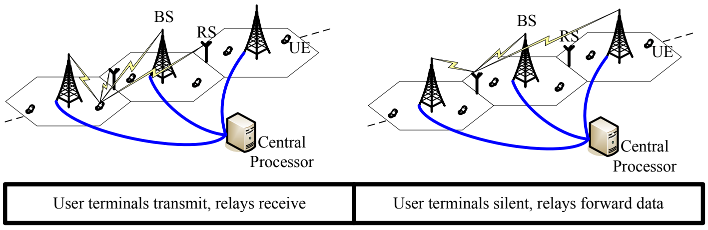

Figure 1: System scenario and two phases of uplink communication

For the wireless communication environment, each channel gain $h_{d,s}$ is composed of two factors: one factor models the distance dependent path loss experienced by the signal and the other factor models the Rayleigh fading. We assume a narrow band single-tap flat (frequency non-selective) Rayleigh fading channel and for this work we ignore the log-normal shadowing effects. The channel gain can be expressed in terms of these two factors as: $h_{d,s} = \psi_{d,s} \times g_{d,s}$. In matrix form this will give:

$$\mathbf{H} = \mathbf{\Psi} \odot \mathbf{G} \tag{2}$$

where $h_{d,s}$ is an element of matrix $\mathbf{H}$ at the $d^{th}$ row and $s^{th}$ column and $\odot$ represents the Hadamard (element-wise) product of two matrices.

The distance dependent path loss factor is modelled using the inverse power law ensuring a minimum separation of reference distance of one meter [5]:

$$\psi_{d,s} = L_0 \left(1 + D_{d,s}\right)^{-\eta} \tag{3}$$

where the constant $L_0$ defines the loss at the reference distance, $D_{d,s}$ is the physical distance between the destination and the reference point at the source and $\eta$ is the path loss exponent. This simplified model fits a wide range of empirical path loss models obtained for various environments by appropriately choosing the parameters $L_0$ and $\eta$. The other factor of the channel gain models the Rayleigh flat fading coefficients, normalised to unit power and circularly symmetric i.i.d. Gaussian: $g_{d,s} \sim \mathcal{CN}(0, 1)$.

A scenario of such a system is depicted in Fig. 1. Each user terminal transmits signal that can be eventually received by several base stations. Since all base stations are connected to a central processor, all received signals are conveyed to the central processor for joint decoding. In the absence of any relay terminals we can obtain an input out equation using the model described above with $N$ base stations as the destinations ($D = N$) and $NK$ total users as the sources ($S = NK$). The sum-capacity of such a system with joint processing at the receiver end is given as [1, 8]:

$$C_{sum} = \mathbb{E}\left(\log_2 \det\left(\mathbf{I}_N + \mathbf{H}\mathbf{Q}_x\mathbf{H}^{\dagger}\mathbf{R}_{\mathbf{w}}^{-1}\right)\right) \tag{4}$$

where the units of the sum-capacity are bits/sec/Hz, $\mathbf{Q}_x$ represents the covariance of the input signals $\mathbf{x}$ and $\mathbf{R}_{\mathbf{w}}$ represents the covariance of the noise vector. Assuming that all inputs are independent we have $\mathbf{Q}_x = P\mathbf{I}$.

In the presence of relays each communication time slot can be considered to have two phases as shown in Fig. 1. We assume orthogonal amplify and forward scheme, as proposed in [6], for the initial investigations. In the first phase of the communication, all users transmit their signals and these signals are received by the relays as well as the base stations. Input output equation for the first phase is given as:

$$\mathbf{y}_1 = \mathbf{H}_{BU}\mathbf{x}_U + \mathbf{w}_{B1} \tag{5}$$

where $\mathbf{y}_1$ is the received vector at the base station array during the first phase, $\mathbf{w}_{B1}$ is the AWGN samples during phase 1 and matrix $\mathbf{H}_{BU}$ is the channel matrix from all the users to all base stations. During this phase, the transmitted signals from the user terminals are given by the vector $\mathbf{x}_U$. During the same phase, the transmitted signals are also received by the relay nodes and this is given as:

$$\mathbf{y}_1^R = \mathbf{H}_{RU}\mathbf{x}_U + \mathbf{w}_R \tag{6}$$

where the channel $\mathbf{H}_{RU}$ is now between the user terminals and the relay node. The AWGN at the relay nodes is given by $\mathbf{w}_R$. In the second phase of communication the signals received at the relays are transmitted to the base stations for joint processing with the signals received in the first phase. The received signal at the base station array during the second phase is given as:

$$\begin{aligned}\mathbf{y}_2 &= \mathbf{H}_{BR}\left[\mu\mathbf{y}_1^R\right] + \mathbf{w}_{B2} &\quad (7)\\ &= \mathbf{H}_{BR}\left[\mu\left(\mathbf{H}_{RU}\mathbf{x}_U + \mathbf{w}_R\right)\right] + \mathbf{w}_{B2} &\quad (8)\end{aligned}$$

Equation (8) is obtained by substituting (6) in (7). The channel from the relay nodes to the base stations is given as $\mathbf{H}_{BR}$ and $\mu$ is the amplification that the relay can provide to the received signals before forwarding them over the next hop. The value of $\mu$ is dependent on the power constraints on the relay node. Assuming that the power is constrained by $P_R$ at the relay nodes, we can derive that for the $j^{th}$ relay we have the constraint

$$\mu^2 \leq \frac{P_R}{PKN\mathbb{E}[h_{r,u}h_{r,u}^\dagger] + \sigma_j^2} \tag{9}$$

In the following section we describe how this model is used to calculate the achievable sum rate and the user rates for the system under consideration.

## 3. Performance Evaluation

For the case where there are no relay stations the maximum sum rate is given by (4). For the case where the relay nodes are also present, we proceed as follows. Since the signals received at the base station arrays during the two phases of communication are jointly decoded we can stack the vectors $\mathbf{y}_1$ and $\mathbf{y}_2$ and by rearranging the terms we can express the stacked vector as follows:

$$\left[\begin{array}{c} \mathbf{y}_1 \\ \mathbf{y}_2 \end{array}\right] = \left[\begin{array}{c} \mathbf{H}_{BU} \\ \mu\mathbf{H}_{BR}\mathbf{H}_{RU} \end{array}\right] \mathbf{x}_U + \left[\begin{array}{c} \mathbf{w}_{B1} \\ \mu\mathbf{H}_{BR}\mathbf{w}_R + \mathbf{w}_{B2} \end{array}\right] \tag{10}$$

which can be more compactly written as (using $\bar{(.)}$ to represent the stacked vector/matrix)

$$\bar{\mathbf{y}} = \bar{\mathbf{H}}\mathbf{x}_U + \bar{\mathbf{w}}. \tag{11}$$

Extending the approach in [9, 6], the maximum achievable sum rate is derived as:

$$R_{OAF} = \frac{1}{2}\log\det\left(\mathbf{I}_{2N} + P\bar{\mathbf{H}}\bar{\mathbf{H}}^{\dagger}\mathbf{R}_{\bar{\mathbf{w}}}^{-1}\right) \tag{12}$$

where the factor of half comes from the two phases used to transmit the signal, $P$ is the per user transmit power constraint and the covariance $\mathbf{R}_{\bar{\mathbf{w}}}$ of the stacked noise vector, $\bar{\mathbf{w}}$, is calculated as

$$\mathbf{R}_{\bar{\mathbf{w}}} = \sigma^2 \left[\begin{array}{cc} \mathbf{I}_N & \mathbf{O}_N \\ \mathbf{O}_N & \mathbf{I}_N + \mu^2\mathbf{H}_{BR}\mathbf{H}_{BR}^{\dagger} \end{array}\right] \tag{13}$$

where $\mathbf{I}_N$ and $\mathbf{O}_N$ represent the identity and an all-zero ($N \times N$) matrix.

Considering the uplink joint decoder, for any given sum rate we can calculate the individual user rate share if we define the specific order in which the user signals will be decoded. Assume that the users are permuted and the permutation is given as $\tilde{\pi}$ with the index of the $i^{th}$ user in the permutation represented as $\tilde{\pi}(i)$. The rate share of any user will be given as

$$r_{\tilde{\pi}(i)} = \log\det\left(\frac{\mathbf{I}_{2N} + P\sum_{j=1}^{i}\bar{\mathbf{h}}_{\tilde{\pi}(j)}\bar{\mathbf{h}}_{\tilde{\pi}(j)}^{\dagger}\mathbf{R}_{\bar{\mathbf{w}}}^{-1}}{\mathbf{I}_{2N} + P\sum_{j=1}^{i-1}\bar{\mathbf{h}}_{\tilde{\pi}(j)}\bar{\mathbf{h}}_{\tilde{\pi}(j)}^{\dagger}\mathbf{R}_{\bar{\mathbf{w}}}^{-1}}\right) \tag{14}$$

where the user at the first position in the permutation ($i = 1$) is decoded last in the decoding process and experiences no interferences since all other users have been decoded and stripped (cancelled) from the aggregate received signal.

## 4. Results and Discussion

We simulate a cellular scenario where $N = 10$ base stations are equally spaced on a system span $D$ (in km). The inter-site distance can be calculated as $D/N$. Users are uniformly randomly distributed on the system span with a density of 10 users/km. Each user has a transmit power constraint of 100 mW and where the relay nodes are used they have a transmit power constraint of 1 W. The path gain at the reference distance of 1m is $L_0 = -38dB$ and the path loss exponent $\eta = 4$ is considered to correspond to empirical models. Noise spectral density $N_0 = -169$ dBm is assumed and a subcarrier of narrow band spectrum ($W = 50$ KHz) is considered where the noise power is calculated as $\sigma^2 = N_0 W$.

In Fig. 2 we compare the uplink sum-rate of the system for various deployment scenarios of a linear multicell joint processing cellular system. The sum rate is plotted against the extent

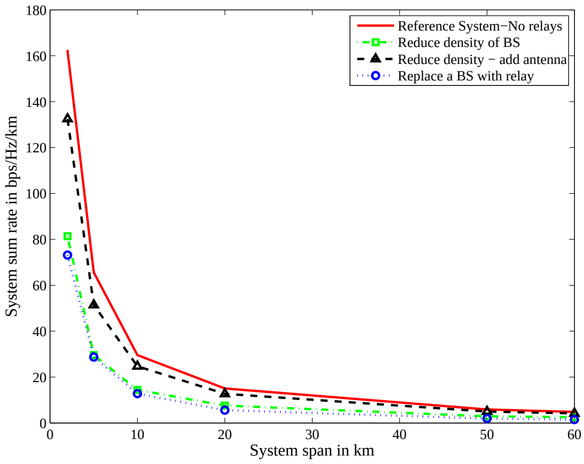

Figure 2: Comparison of spectral efficiency as a function of base station density of the cellular deployment for different scenarios

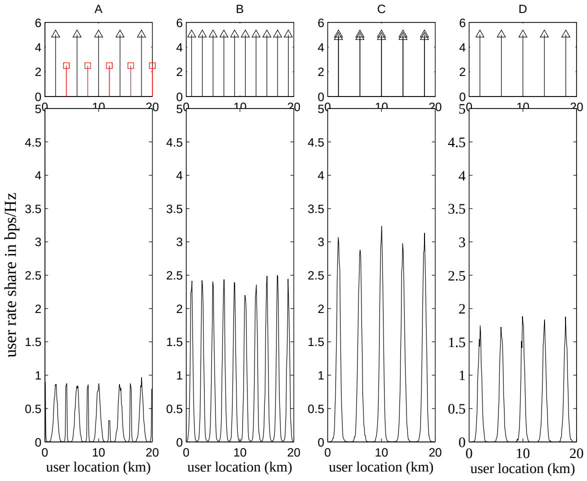

Figure 3: Average user rate share as a function of user location on the cellular grid along with the deployment map of base stations and relays

of the cellular system while the total number of cells within this extent is held fixed to 10 base stations. The sum-rate is normalised (given in bps/Hz per km) to be able to compare different cellular extents. It can be observed that for all systems compared, the spectral efficiency per unit distance is decreased when larger cellular systems are covered with the fixed number of base stations (in other words the density of the base stations is reduced).

A benchmark system deployment of a linear cellular array is considered and its sum rate is calculated using (12). The density of the base stations is decreased by removing alternate base stations and sum-rate is calculated using the same equation but with a reduced number of receivers (and a smaller channel matrix). As seen in Fig. 2 the sum rate is approximately halved. In the next scenario the removed base station is compensated with an additional antenna at the closest base station. This recovers the loss in sum-rate to a large extent. Finally the removed base station is replaced by a relay node and it is observed that approximately the same sum-rate is achieved as in the case where the base station density was halved. This suggests that for this scenario the deployment of relay nodes is not beneficial, at least from sum-rate point of view, however the results in Fig. 3 provide some deeper insights.

In Fig. 3 we show the average rate share of each user as a function of their position in the cellular system. We use random decoding order at the joint processor to calculate the user rate share for each system snapshot using (14) and evaluations of 1000 snapshots is averaged to plot the results. All four scenarios are plotted side-by-side in the figure. The reference system is shown in (B) with the top figure indicating the position of the base stations (bars with triangular marker at the top end). It can be observed that the users close to the base stations are getting the larger share of the sum-rate while the users at the boundaries of the coverage of the two cells get very low rate-share. When alternate base stations are removed, we obtain the rate distribution as shown in (D). In comparison to the reference system in (B) the peak rates are reduced as well as the rates at the cell-edges. A larger number of users at the cell-edges get very low rate. Adding an additional antenna (indicated by an extra triangular marker in the base station position indicators of the top subfigure under column C) compensates for the loss in sum rate and it can be observed that higher peak rates, as compared to the reference system in (B), are achieved but the share of the rate for the edge users is much smaller. In column (A), the relay nodes are introduced to compensate for the removed base station. The position of the relay nodes is indicated with smaller bars and a square marker. The system has smaller peak rates but the users close to the relay nodes get a fairer share from the sum-rate. In short, the benefit of relay nodes in this multicell joint processing scenario is not in terms of sum-rate but is in terms of a better service for the cell-edge users.

## 5. Conclusion and Future Work

We have considered a joint processing cellular system's uplink and compared the performance of this system for different deployment scenarios. A linear cellular system is considered as a benchmark and the effect of various changes to this benchmark deployment are analytically studied. Information theoretic sum-rate and user rate shares are derived and calculated for the changed scenarios. It is observed that decreasing the density reduces the sum rate and makes user rate shares relatively unfair. Replacing a base station with an additional antenna recovers large part of the loss in sum-rate due to reduced base stations but user rate share for the users at the edges of the cell is reduced. By replacing base stations by the relays and using the amplify and forward scheme the sum-rate is same as for the case where alternate base stations are removed. However the introduction of the relays provides a relatively fair user rate distribution. The work suggests that the relay nodes can play an important role to reshape the user rate shares in the cellular environment even in the case where the base stations are fully cooperating for the decoding of the uplink signals.

As part of the future work, we are studying the problem for the planar cellular arrays with more detailed propagation models that include the log-normal shadow fading as well. In addition to amplify and forward relaying strategy other strategies like decode and forward and compress and forward will also be investigated. Effect of multiple antennas on the relay node and user terminal is also under investigation. Finally, considering the role of relays for the downlink channel will also be a useful direction for the further work.

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