Performance evaluation Of HARQ schemes for Cooperative Regenerative Relaying

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Abstract—.**Two Hybrid ARQ (HARQ) schemes based on selective decode and forward are considered for a cooperative half-duplex relay channel. Two time slot types: T_1 slot for relay listening and T_2 slot for relay forwarding are assumed to accommodate half duplex relaying. The considered HARQ schemes differ in the frequency of the ARQ feedback: one is frame and the other is slot based where each frame is composed of one T_1 followed by one T_2 slot. Two types of encodings: repetition coding (RC) and unconstrained coding (UC) (also known as incremental redundancy) are assumed. Outage performance analysis is carried out for both RC and UC. The state transition models of the considered protocols are presented and are used to analytically calculate the HARQ throughput and latency performance; thus avoiding time consuming Monte Carlo based evaluations. The provided analysis enables us to predict the system performance and tune its transmission parameters (transmission rate and frame structure) for any combination of the signal to noise ratio (SNR) of the constituent links.**

Index Terms— **HARQ, outage, Relay Channels, cooperative, DF**

I. Introduction

Cooperative communication tries to exploit idle radio nodes in the vicinity of the sending or receiving nodes by establishing a proper transmission/reception strategy to further improve the system performance. One of the common strategies called decode and forward (DF) [1] is a form of regenerative relaying where the helping relay node decode, re-encode, and then forward the received message to the final destination. The destination node combines its observations on the directly transmitted signal and the forwarded signal to obtain a reliable decoding of the main message. It is likely that under poor and static fading channel conditions the transmitted data packet dose not go through in the first attempt despite of being supported by cooperative relaying. In the case of such an event data packets needs to be retransmitted under an appropriate *Automatic Retransmission reQuest* (ARQ) procedure. This can be further improved by combining channel coding and ARQ that is commonly known as Hybrid ARQ (HARQ). Common encoding techniques used in conjunction with HARQ are *i*) repetition coding (RC) and *ii*) unconstrained coding (UC) also known as incremental redundancy (IR). Application of HARQ techniques over multi-hop and cooperative realying has been considered in [3] and [4]. Reference [3] considers a multiple relay network and addresses the scheduling problem of determining when each node should transmit. This reference resorts to Monte Carlo based analysis to derive the considered system performance. Reference [4], on the other hand focuses on a single relay cases and drives the diversity-multiplexing-delay trade-off under asymptotic signal to noise ratio (SNR) condition. Here we focus on single relay case and derive the outage performance for both RC and UC coding schemes and for any range of SNRs. Two HARQ protocols are considered that differ in the ARQ feedback rate. The state transition models of the considered protocols are presented and are used to analytically calculate the HARQ throughput and latency performance. The provided analysis enables us to predict the system performance and tune its transmission parameters for any combination of the signal to noise ratio (SNR) of the constituent links.

II. System Model

Let assume a three-node cooperative communication system composed of: source node S, relay node R, and destination node D. Let also impose the practical constraint of half duplex relay system, i.e. the relay node is only able to receive or transmit at a given time. In a system not performing any ARQ mechanism communication towards destination could be accomplished in two phases: in the first phase S broadcasts to both R and D, and in the second phase if R has successfully decoded the message, it will join S in its transmission towards D. This will result a frame structure composed of two time slots as depicted in Figure 1. The durations of the time slots need not to be the same and we define the duplexing ratio α as the percentage of the first time slot (denoted by T_1) in the whole frame duration. As also shown in figure 1 we enumerate links S-R, S-D, and R-D with 0, 1, and 2, respectively. Under static and poor fading conditions the considered system might not be able to successfully deliver the data message to the destination and thus provision of an error control mechanism through preferably an HARQ process will be necessary. The HARQ process will initiate necessary retransmissions until the data packet is correctly delivered to the destination. Here we make following assumptions about the engaged HARQ process:

- *Limited number of transmissions*: To avoid inefficiency due to large number of transmissions in poor channel conditions, the transmissions carried out by S and R are limited to some finite number(s).

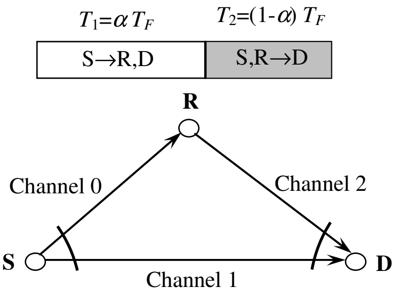

Figure 1 A Cooperative communication model composed of a source, destination, and single relay.

- *HARQ Feedback*: the acknowledged (ACK) or not acknowledged (NAK) feedback is sent by the destination and is received error-free by both S and R. The ACK/NAK feedback frequency could be per time slot or per frame.
- *Relay Node Forwarding*: The relay node will automatically benefit from HARQ based transmissions of S and will combine all the signals received from S to improve its decoding. R will participate in transmission of the message if it has successfully decoded the message.
- *Two-phased Frame Structure*: A two-phased frame structure composed of one T_1 slot followed by one T_2 slot has to be preserved, where R can only listen in T_1 and forward in T_2 slots. A typical example justifying this case is when R is a fixed relay node deployed by the network and is shared for forwarding several users' traffic. In this case R also has to listen in T_1 slots to other traffics that have been multiplexed through any multiple access scheme.

Based on the above assumptions two following protocols can be adopted:

1. *Protocol A*: ACK/NAK feedback from D is provided on frame basis and R is only allowed to transmit signal in the T_2 slots. The number of transmission frames is limited to $N1$.
2. *Protocol B:* Similar to protocol A, but in this case the ACK/NAK feedback from D is per time slot. Thus by reception of ACK at T_1, S and R will stop their transmission and will not enter to T_2 phase. However the two-phased frame structure will be maintained.

III. Signal Model

Here we assume a static fading condition for the signal transmission. The message $w\in\{1,2,\ldots,2^{M\rho}\}$ is attempted to be delivered by HARQ process to the destination node, where M is the length of the frame in number of samples, and ρ is the transmission rate. Let $\underline{\boldsymbol{x}}_{s,k}^{j}(w)$ denote the signal code sequence transmitted by S at transmission k and slot T_j (j=1, 2). Let also $\underline{\boldsymbol{x}}_{r,k}^{2}(w)$ denote the signal sequence transmitted by R at transmission k and slot T_2. Assuming static flat (non frequency selective) fading channel for all the links, the received signal sequences at R and D during the HARQ process can be written as follows:

$$\underline{\boldsymbol{y}}_{r,k}=h_{0,k}\sqrt{\mu_0}\,\underline{\boldsymbol{x}}_{s,k}^{1}(w)+\underline{\boldsymbol{v}}_{r,k}, \tag{1}$$
$$\underline{\boldsymbol{y}}_{k}^{1}=h_{1,k}\sqrt{\mu_1}\,\underline{\boldsymbol{x}}_{s,k}^{1}(w)+\underline{\boldsymbol{v}}_{k}^{1}$$
$$\underline{\boldsymbol{y}}_{k}^{2}=h_{1,k}\sqrt{\mu_1}\,\underline{\boldsymbol{x}}_{s,k}^{2}(w)+h_{2,k}\sqrt{\mu_2}\,\underline{\boldsymbol{x}}_{r,k}^{2}(w)+\underline{\boldsymbol{v}}_{k}^{2}$$

In the above expressions superscripts 1 and 2 denote the slots T_1 and T_2., respectively. The subscript k denotes the k-th HARQ transmission, and finally subscripts s and r denote the source and relay nodes. μ_l is the average signal to noise ratio of the link l=0,1,2. Also the fading coefficients for different links l and transmissions k are denoted by $h_{l,k}$. we assume that $h_{l,k}$ are random unit-power with complex Gaussian distribution and are mutually independent for different l and k. The independence along k means that either HARQ transmissions are spaced with enough gaps along the time or are using different scheduled frequency resources. The received signal sequences are denoted by $\underline{\boldsymbol{y}}_{r,k}$, $\underline{\boldsymbol{y}}_{k}^{1}$, and $\underline{\boldsymbol{y}}_{k}^{2}$, that are received by R, and D in time slots 1 and 2 and transmission k, respectively. The corresponding additive noise sequences are denoted by $\underline{\boldsymbol{v}}_{r,k}$, $\underline{\boldsymbol{v}}_{k}^{1}$, and $\underline{\boldsymbol{v}}_{k}^{2}$. All the components of the noise sequences are assumed to be zero-mean unit-power circularly-symmetric and white Gaussian. The instantaneous receive SNR at k-th transmission for link l will be $\zeta_{l,k}=|h_{l,k}|^2\mu_l$ that will have an exponential distribution with mean μ_l: $\zeta_{l,k}\sim E(\mu_l)$.

We assume two HARQ encoding methods:

- *Repetition Coding* (RC): $T_1=T_2$ and all the code sequences transmitted by S are identical and the code sequences transmitted by R are Alamouti space-time coded versions of the source sequences:
$$\underline{\boldsymbol{x}}_{s,k}^{1}(w)=\underline{\boldsymbol{x}}_{s,k}^{2}(w)=\underline{\boldsymbol{x}}_{s}(w)\quad\forall k.$$
$$\underline{\boldsymbol{x}}_{r,k}^{2}(w)=ST\left(\underline{\boldsymbol{x}}_{s}(w)\right)\quad\forall k$$
where $ST\left((a_1,a_2,a_3,a_4,\ldots)\right)=\left(-a_2^*,a_1^*,-a_4^*,a_3^*,\ldots\right)$
The both of R and D receivers will do the maximal ratio (also known as Chase) combining of the different received copies before feeding them to their decoder.
- *Unconstrained Coding* (UC): T_1 and T_2 are not necessarily the same and all the transmitted sequences by either of S and R are different. All the transmitted sequences will form a longer code sequence, the corresponding code book will be the used for decoding at both R and D.

In the next sections we provide performance analysis for the above two encodings. The analysis will be under the basic assumption of long enough codes capable of achieving the realized capacity limit. Thus we resort to capacity outage analysis and then drive the performance of the considered HARQ protocols.

IV. Outage Analysis

In this section we drive the outage performance for arbitrary number of transmissions by S and R. The analysis is based on two basic assumptions:

- *Capacity achieving error correction code*
- *Gaussian input signal*: The components of the transmitted code sequences are complex Gaussian distributed and are not constrained to any practical finite alphabet such as QAM.

The above assumptions allow us to take an analytical approach and drive performance bounds for any kind of practical error correction coding and modulation. For the sake of exposition let's only focus on protocol B described in previous section. Similar approach will be applicable to the other protocol. Let $C_{n1,n2}$ denotes the realized overall capacity after $n1$ transmissions by S and $n2$ transmissions by R, where given $n1=2m+t$ with $0\leq t\leq 1$, S has $m+t$ transmissions using T_1 slots and m transmissions using T_2 slots. The transmissions of R will coincide with $n2$ last T_2 slots transmissions of the S (obviously $n2\leq m$). The corresponding realized instantaneous SNRs with theses transmissions will be respectively: 1) Source T_1 slot transmissions: $\{\zeta_{0,1},\ldots, \zeta_{0,m+t}\}$ and $\{\zeta_{1,1},\ldots, \zeta_{1,m+t}\}$ (associated to links 0 and 1, respectively); 2) Source T_2 slot transmissions: $\{\zeta_{1,1},\ldots, \zeta_{1,m}\}$; and 3) Relay T_2 slot transmissions: $\{\zeta_{2,m-n2+1},\ldots, \zeta_{2,m}\}$. $C_{n1,n2}$ for the two types of RC and UC encoding will be as follows:

$$\text{RC: } C_{n1,n2}=\frac{1}{2}I\left(Z_{n1,n2}\right) \text{ with} \tag{2}$$

$$Z_{2m,n2}=2\sum_{k=1}^{m}\zeta_{1,k}+\sum_{k=m-n2+1}^{m}\zeta_{2,k} \text{ and}$$

$$Z_{2m+1,n2}=Z_{2m,n2}+\zeta_{1,m+1}$$

$$\text{UC: } C_{2m,n2}=\alpha\sum_{k=1}^{m}I\left(\zeta_{1,k}\right)+$$

$$\left(1-\alpha\right)\sum_{k=m-n2+1}^{m}I\left(\zeta_{1,k}+\zeta_{2,k}\right)$$

$$C_{2m+1,n2}=C_{2m,n2}+\alpha I\left(\zeta_{1,m+1}\right)$$

The above capacity expressions are normalized to the frame duration M. The term $I(\zeta)=\log_2(1+\zeta)$ is the maximized mutual information for a single-antenna transmission with complex Gaussian input. The joint S and R transmission using Alamouti coding in RC case will result in addition of the corresponding SNRs, and their joint UC transmissions with instantaneous SNRs ζ_1 and ζ_2 will result in a multiple-input single-output channel with capacity $I(\zeta_1+\zeta_2)$. The terms $\zeta_{1,k}$ and $\zeta_{2,k}$ are independent and exponentially distributed random variables with means μ_1 and μ_2, respectively. As a result the terms $Z_{n1,n2}$ and $C_{n1,n2}$ will be random, and the outage may happen that is defined as follows:

$$F_{n1,n2}\left(\rho\right)=\Pr\left\{C_{n1,n2}<\rho\right\} \tag{3}$$

Alternatively the outage for RC case can be expressed as follows:

$$F_{n1,n2}\left(\rho\right)=\Pr\left\{Z_{n1,n2}<\zeta_\rho\right\} \text{ where } \zeta_\rho=2^{2\rho}-1 \tag{4}$$

To be able to calculate the outage probability we first derive the characteristic function of the random variables $Z_{n1,n2}$ and $C_{n1,n2}$:

$$\text{RC: } \Psi_{n1,n2}\left(s\right)=E\left[e^{-sZ_{n1,n2}}\right], \tag{5}$$

$$\text{UC: } \Phi_{n1,n2}\left(s\right)=E\left[e^{-sC_{n1,n2}}\right],$$

where $E[X]$ means expectation with respect to random variable X. Considering the independence of $\zeta_{1,k}$ and $\zeta_{2,k}$ and their exponential distribution the above function can be expressed as follows:

$$\Psi_{2m,n2}\left(s\right)=\frac{1}{\left(1+2\mu_1 s\right)^m}\cdot\frac{1}{\left(1+\mu_2 s\right)^{n2}}, \text{ and} \tag{6}$$

$$\Psi_{2m+1,n2}\left(s\right)=\frac{1}{1+\mu_1 s}\Psi_{2m,n2}\left(s\right)$$

$$\Phi_{n1,n2}\left(s\right)=\left[\varphi_1\left(\alpha s\right)\right]^{\lceil n1/2\rceil-n2}\left[\varphi_{12}\left(s;\alpha\right)\right]^{n_2}, \text{ where}$$

$$\varphi_{12}\left(s;\alpha\right)=E\left[e^{-s\left(\alpha I\left(\zeta_1\right)+\left(1-\alpha\right)I\left(\zeta_1+\zeta_2\right)\right)}\right] \text{ and}$$

$$\varphi_1\left(s\right)=E\left[e^{-sI\left(\zeta_1\right)}\right] \text{ with } \zeta_1\sim\mathrm{E}(\mu_1) \text{ and } \zeta_2\sim\mathrm{E}(\mu_2)$$

Computing the inverse of $\Psi_{n1,n2}\left(s\right)$ through decomposition to partial fractions will lead us to the probability distribution function (pdf) of $Z_{n1,n2}$ which will take the form of a composite Gamma distribution, i.e. weighted summation of a number of Gamma pdfs with different orders. The calculation of outage probability through the derived pdf will be a straight forward job. However calculation of the functions $\varphi_1\left(s\right)$ and $\varphi_{12}\left(s;\alpha\right)$ is difficult and we propose to resort to numerical calculation for a predefined sets of values for s and α. Then using (6) $\Phi_{n1,n2}\left(s\right)$ can be calculated. To calculate the outage probability we can resort to Laplace inversion formula [5]:

$$\text{UC: } F_{n1,n2}\left(\rho\right)=\frac{1}{2\pi j}\int_{a-j\infty}^{a+j\infty}\frac{\Phi_{n1,n2}\left(s\right)e^{s\rho}}{s}ds, \tag{7}$$

where a belongs to the intersection of the region of the convergence of $\Phi_{n1,n2}\left(s\right)e^{s\rho}$ with the real positive line. Above integral then can be calculated using a method based on Gauss-Chebyshev quadrature rule [7]. The advantage of this method is that it will require the knowledge of a limited predetermined values of s taken on $\Phi_{n1,n2}\left(s\right)$.

For relay outage we define $C_{n1}^{(0)}$ as the accumulated capacity after $n1$ transmission from S in T_1 slots. Similarly the following outage probability is defined:

$$F_{n1}^{(0)}\left(\rho\right)=\Pr\left\{C_{n1}^{(0)}<\rho\right\} \tag{8}$$

The derivation performed above can be repeated for the other protocol as well as for the relay outage. Table 1 summarises all the capacity terms $C_{n1}^{(0)}$ and $C_{n1,n2}$ for the considered HARQ protocols.

V. HARQ Performance Analysis

In this section the throughput, latency, and last transmission outage probability that are the main HARQ performance parameters are derived. To do the analysis we drive the state transition model of the considered HARQ processes.

Table 1 Accumulated Capacity $C_{n1,n2}$ and $C_{n1}^{(0)}$

		**Protocols A and B ($n1=2m$)
*RC:	$Z_{n1,n2}$	$2\sum_{k=1}^{m}\zeta_{1,k}+\sum_{k=m-n2+1}^{m}\zeta_{2,k}$
	$Z_{n1}^{(0)}$	$\sum_{k=1}^{n1}\zeta_{0,k}$
UC:	$C_{n1,n2}$	$\alpha\sum_{k=1}^{m}I\left(\zeta_{1,k}\right)+(1-\alpha)\sum_{k=m-n2+1}^{m}I\left(\zeta_{1,k}+\zeta_{2,k}\right)$
	$C_{n1}^{(0)}$	$\alpha\sum_{k=1}^{n1}I\left(\zeta_{0,k}\right)$

*$C_{n1,n2}=I(Z_{n1,n2})/2$ and $C_{n1}^{(0)}=I(Z_{n1}^{(0)})/2$

**RC: $Z_{2m+1,n2}=Z_{2m,n2}+\zeta_{1,m+1}$

UC: $C_{2m+1,n2}=C_{2m,n2}+\alpha I(\zeta_{1,2m+1})$

We define state $\mathcal{S}_{n1,n2}$ for the considered protocols as follows:

***Protocol A*:** D decodes successfully after $n1$ and $n2$ slot T_2 transmissions from the source and relay nodes, respectively. This means S has performed overall $2n1$ transmissions half using slot T_1 and half using slot T_2.

***Protocol B*:** D decodes successfully after $n1$ and $n2$ transmissions from the source and relay nodes, respectively. Compared to protocol A this time $n1$ is meant for overall number of S transmissions in both T_1 and T_2 slots.

Except state $\mathcal{S}_{0,0}$ each state $\mathcal{S}_{n1,n2}$ will also have a corresponding state $\bar{\mathcal{S}}_{n1,n2}$ with similar number of transmissions but no success in decoding by D. The resulted state transition diagrams are depicted in Figure 2. Also shown are the state transition probabilities. Here we drive these probabilities for only protocol B, and leave the derivation of the protocol A to interested reader. For this protocol following state transitions can happen:

1. $\bar{\mathcal{S}}_{n1-1,0}\rightarrow\bar{\mathcal{S}}_{n1,0}$ and $\bar{\mathcal{S}}_{n1-1,0}\rightarrow\mathcal{S}_{n1,0}$ with probabilities $p_{n1,0}$ and $q_{n1,0}$, respectively.
2. $\bar{\mathcal{S}}_{2m-1,n2-1}\rightarrow\bar{\mathcal{S}}_{2m,n2}$ and $\bar{\mathcal{S}}_{2m-1,n2-1}\rightarrow\mathcal{S}_{2m,n2}$ with probabilities $p_{2m,n2}$ and $q_{2m,n2}$, respectively.
3. $\bar{\mathcal{S}}_{2m,n2}\rightarrow\bar{\mathcal{S}}_{2m+1,n2}$ and $\bar{\mathcal{S}}_{2m,n2}\rightarrow\mathcal{S}_{2m+1,n2}$ with probabilities $p_{2m+1,n2}$ and $q_{2m+1,n2}$, respectively.

These transition probabilities can be calculated using different outage event probabilities. As an example let's focus on $p_{2m,0}$. This probability corresponds to R and D to be in outage after $m\times T_1$ and $m\times T_2$ transmissions given that both R and D were in outage after $m\times T_1$ and $(m\text{-}1)\times T_2$ transmissions:

$$\begin{aligned}p_{2m,0}&=\Pr\left\{C_m^{(0)}<\rho,C_{2m,0}<\rho\middle|C_{m-1}^{(0)}<\rho,C_{2m-1,0}<\rho\right\}\\&=\Pr\left\{C_m^{(0)}<\rho\middle|C_{m-1}^{(0)}<\rho\right\}\Pr\left\{C_{2m,0}<\rho\middle|C_{2m-1,0}<\rho\right\}\\&=\frac{F_m^{(0)}(\rho)}{F_{m-1}^{(0)}(\rho)}.\frac{F_{2m,0}(\rho)}{F_{2m-1,0}(\rho)}.\end{aligned}\tag{9}$$

where the second line comes from the independence of R and D outage events, and the last line comes from the property that for events $E_1\subseteq E_2$ the conditional probability $\Pr\{E_1|E_2\}$ will reduce to $\Pr\{E_1\}/\Pr\{E_2\}$. Initial values $F_0^{(0)}(\rho)=1$ and $F_{0,0}(\rho)=1$ are assumed. Taking the similar approach the other transition probability can also be calculated:

$$q_{2m,0}=\frac{F_m^{(0)}(\rho)}{F_{m-1}^{(0)}(\rho)}.\left(1-\frac{F_{2m,0}(\rho)}{F_{2m-1,0}(\rho)}\right),\tag{10}$$

$$p_{2m,1}=\left(1-\frac{F_m^{(0)}(\rho)}{F_{m-1}^{(0)}(\rho)}\right).\frac{F_{2m,1}(\rho)}{F_{2m-1,0}(\rho)},$$

$$q_{2m,1}=\left(1-\frac{F_m^{(0)}(\rho)}{F_{m-1}^{(0)}(\rho)}\right).\left(1-\frac{F_{2m,1}(\rho)}{F_{2m-1,0}(\rho)}\right),$$

$$p_{2m,n2}=\frac{F_{2m,n2}(\rho)}{F_{2m-1,n2-1}(\rho)}\text{ and }q_{2m,n2}=1-p_{2m,n2}\quad\forall n2>1$$

$$p_{2m+1,n2}=\frac{F_{2m+1,n2}(\rho)}{F_{2m,n2}(\rho)}\text{ and }q_{2m+1,n2}=1-p_{2m+1,n2}\quad\forall n2$$

Using these transition probabilities it will be a straight forward practice to derive the state probabilities $Q_{n1,n2}=\Pr\left\{\mathcal{S}_{n1,n2}\right\}$ and $\bar{Q}_{n1,n2}=\Pr\left\{\bar{\mathcal{S}}_{n1,n2}\right\}$:

$$Q_{n_1,0}=q_{n_1,0}\bar{Q}_{n_1-1,0}\text{, and }\bar{Q}_{n_1,0}=p_{n_1,0}\bar{Q}_{n_1-1,0}\tag{11}$$

For $n_2\geq1$:

$$Q_{2m,n_2}=q_{2m,n_2}\bar{Q}_{2m-1,n_2-1}\text{, and }\bar{Q}_{2m,n_2}=p_{2m,n_2}\bar{Q}_{2m-1,n_2-1}$$

$$Q_{2m+1,n_2}=q_{2m+1,n_2}\bar{Q}_{2m,n_2}\text{, and }\bar{Q}_{2m+1,n_2}=p_{2m+1,n_2}\bar{Q}_{2m,n_2}$$

Using above relations the state probabilities can be recursively calculated starting with the initial value of $\bar{Q}_{0,0}=1$. Now we can apply the renewal-reward theorem [8] to drive the throughput and latency of the protocol. The HARQ process will end by either entering states $\mathcal{S}_{n1,n2}$ that will lead to a successful delivery of data with rate reward of ρ, airtime of τ_{n1} and latency of d_{n1}, or will be forced to terminate at states $\bar{\mathcal{S}}_{N1,n2}$ with no rate reward and resulted airtime of τ_{N1}, and latency of d_{N1}, where τ_{n1} and d_{n1} are normalized to the frame duration M and are $\tau_{n1}=d_{n1}=m$ for $n1=2m$, and $\tau_{n1}=m+\alpha$, $d_{n1}=m+1$ for $n1=2m+1$. The average airtime $E[\mathcal{T}]$, latency $E[\mathcal{D}]$, and rate reward $E[\mathcal{R}]$ are derived by averaging over the aforementioned states:

$$E[\mathcal{T}]=\sum_{n1=1}^{2N1}\sum_{n2=0}^{\lfloor n1/2\rfloor}Q_{n1,n2}\tau_{n1}+\sum_{n2=0}^{N1}\bar{Q}_{2N1,n2}\tau_{2N1}\tag{12}$$

$$E[\mathcal{D}]=\sum_{n1=1}^{2N1}\sum_{n2=0}^{\lfloor n1/2\rfloor}Q_{n1,n2}d_{n1}+\sum_{n2=0}^{N1}\bar{Q}_{2N1,n2}d_{2N1}$$

$$E[\mathcal{R}]=\rho\sum_{n1=1}^{2N1}\sum_{n2=0}^{\lfloor n1/2\rfloor}Q_{n1,n2}\text{, and }P_{out}=\sum_{n2=0}^{N1}\bar{Q}_{2N1,n2}$$

Also provided is the residual outage probability P_{out} that corresponds to the forced to terminate states. The average throughput of the protocol denoted by η (ρ,α) will be the ratio of average rate reward to the average airtime [8]:

$$\eta(\rho,\alpha)=\frac{E[\mathcal{R}]}{E[\mathcal{T}]}\tag{13}$$

The resulted throughput is a function of the transmission rate ρ and the duplexing ratio α, where for RC case only the value of α=0.5 is supported. Therefore, the throughput of the system can be optimized by proper selection of ρ and α:

$$\eta_{opt} = \max_{\rho,\alpha} \eta(\rho,\alpha) \tag{14}$$

Performance of protocol A can also be derived through the same practice of the first calculation of the state transition probabilities, and then calculation of the state probabilities. Here due to the lack of space we only provide the final results:

Protocol A:

$$E[\mathcal{D}] = E[\mathcal{T}] = \sum_{n1=1}^{N1}\sum_{n2=0}^{n1} Q_{n1,n2}\tau_{n1} + \sum_{n2=0}^{N1} \bar{Q}_{N1,n2}\tau_{N1}, \tag{15}$$

with τ_{n1}=$n1$,

$$E[\mathcal{R}] = \rho \sum_{n1=1}^{2N1}\sum_{n2=0}^{\lfloor n1/2 \rfloor} Q_{n1,n2}, \text{ and } P_{out} = \sum_{n2=0}^{N1} \bar{Q}_{N1,n2}.$$

Equations (13) and (14) will be used to compute the average throughput of theses protocols. Next section is devoted to some numerical results and performance comparisons.

VI. Numerical and Simulation Results

Here we provide some simulation and analytical results for the considered HARQ protocols. The analytical derivations are further verified by extensive Monte Carlo based simulations. The results in this section are provided for the maximum number of 4 frame transmissions. Figure 3 compares the throughput performance of the two protocols with two possible RC and UC encoding for a certain SNR configuration of the three constituent node-to-node links. The SNR of the link 1 (S-D link) is used as the reference and the SNR of the two other links are adjusted relative to it: μ_0=δ_0. μ_1 and μ_2=δ_2. μ_1, where δ_0 and δ_2 represent the SNR offsets of the links 0 and 2, respectively. The duplexing ratio of α=0.3, 0.5, and 0.7 and transmission rate of ρ=1 are used. Due to the per-frame ARQ operation at high SNR regime protocol A attains half the throughput of the protocol B. The UC encoding performs better than RC for all the protocols especially at low to moderate SNRs. The comparison made in this figure is not completely fair as the throughput of each protocol must be first optimized with respect to ρ and α. In this regard Figure 4 is providing better insight and more fair comparison. This figure provides 3D surfaces for throughput gain of the cooperative protocols with respect to the optimized throughput of direct S-D communication. The HARQ protocol of the direct case was also limited to 4 transmissions. Optimisation for each protocol was carried out over a large range of transmission rates with a fine level of 0.1 bit/use granularity for ρ and 0.1 granularity for α. S-R SNR μ_0=10 dB is used and the two other links SNRs are allowed to change over a large range of values. As it is observed all the cooperative protocols has gain in low γ_1 and moderate to large γ_2 region. In this region protocols A and B has almost same performance for both types of RC and UC encodings. As γ_1 starts increasing both protocols start losing their gain. RC encoding results in a large negative gain especially for protocol A. This is due to repetitive structure of the RC code within a frame (S transmits same codeword over T_1 and T_2 slots). Protocol A with UC encoding has zero gain in large γ_1 region while protocol B performs better than direct links thanks to its per slot ARQ feedback and employed UC encoding. Finally Figure 5 provides the throughput-optimizing transmission rate for the direct transmission as well as the considered cooperative protocols. The results are only for RC encoding and similar trend of behavior is observed for UC encoding. As it observed the optimum transmission rate depends on the SNR configuration of the constituent links. Therefore; throughput optimization requires the knowledge on average SNR of all the links. Provision of this knowledge is not practically difficult and requires reporting the measured average SNRs from the relay and destination nodes to the source node. The availability of this knowledge will also allow efficient use of

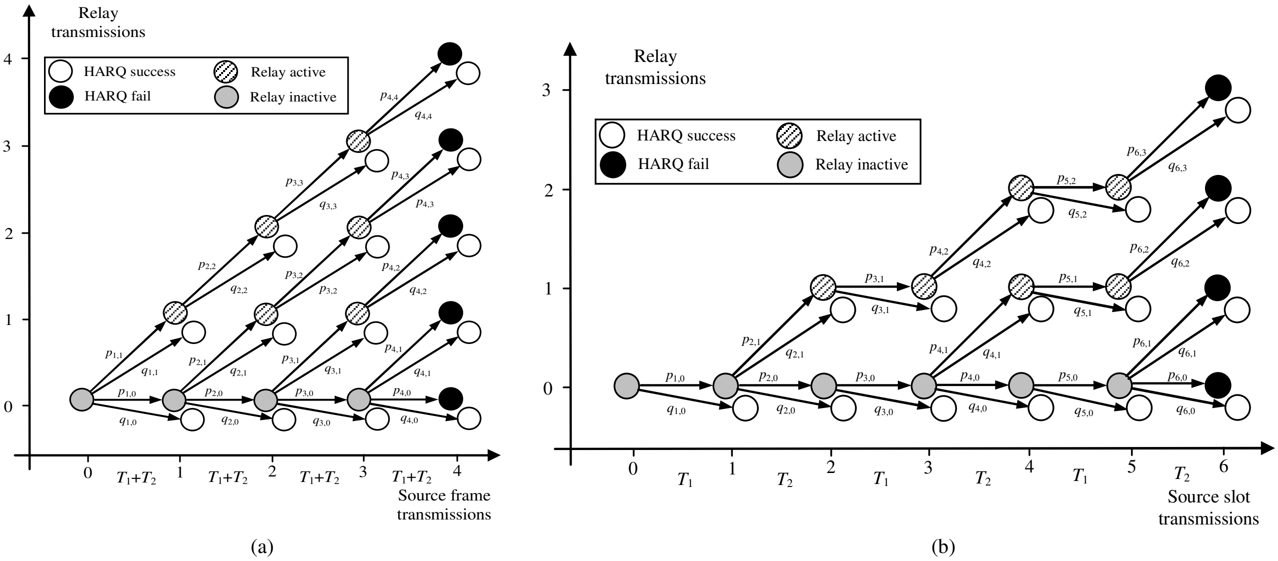

Figure 2 HARQ state transition diagram for (a) protocol A, and (b) protocol B.

the considered protocols in the SNR regions that they deliver gain over direct communication. In this regard protocol A is not in disadvantage with respect to protocol B as it requires less ACK/NAK feedback while attaining almost the same performance as protocol B in low S-D and moderate to high R-D SNRs.

VII. Conclusion

In this paper two typical HARQ schemes are considered for a cooperative communication system with single regenerative relay. The proposed protocols are based on the assumption of a half-duplex relay shared by several traffics. Therefore, a frame structure composed of two time slots: T_1 and T_2 is assumed, where relay is only allowed to forward at certain time slots (T_2 slots). Outage and throughput performance measures are derived for both repetition and unconstrained (incremental redundancy) codings. The presented analysis allows accurate evaluation of DF based cooperative HARQ protocols without resorting to time consuming Monte Carlo based evaluation approaches. This will allow identification of the protocol throughput gain over the direct communication and engage right protocol at right situation. Also the presented analysis allows us to accurately calculate the required transmission rate and frame structure that optimizes the throughput. Future work could be on the extension of the analysis to cases where relay is another user terminal with no restriction to follow two-phased frame structure. Also extension of the analysis to hybrid approach of amplify/decode and forward (AF/DF) is under study. Finally extension to multiple relays and nodes with multiple antennas is expected.

VIII. Acknowledgment

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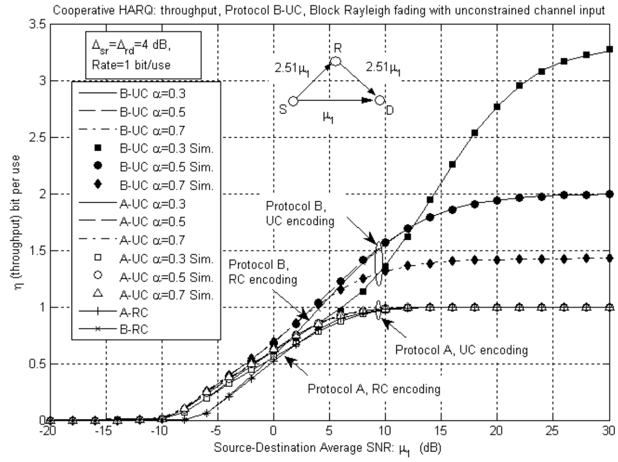

Figure 3 Cooperative HARQ throughput performance: for the two considered protocols, UC (α=0.3, 0.5, 0.7) and RC (α=0.5) encoding.

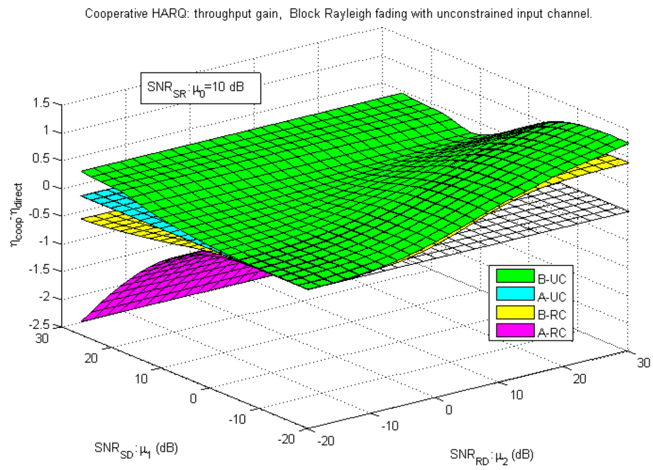

Figure 4 Cooperative HARQ throughput performance gain over direct communication for the considered protocols and UC/RC encoding.

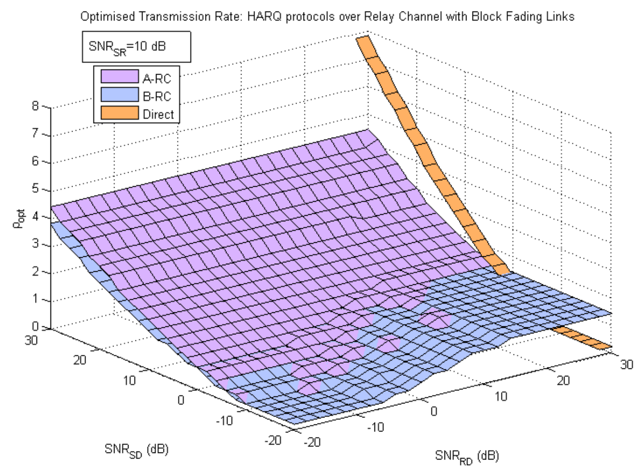

Figure 5 Cooperative HARQ optimum transmission rate for the direct communication and the considered protocols and RC encoding.