Optimum Decoding of Full Decode and Forward Scheme over Cooperative Relay Channels

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***Abstract*—Optimum decoding of the cooperative full decode and forward (DF) scheme is derived. It is shown that optimum decoding requires marginalization of a so called partially observed likelihood function over all relay decoder outcomes. Bhattacharyya bound on relay decoding error is used to fully factorize the partially observed likelihood function. The full factorization will allow efficient marginalization over the joint source-relay code space. Analytical tight error bound is derived that its computation further requires efficient tools to compute weight structure of the joint source-relay code. For a simple (7,4) Hamming code under fast fading condition the derived optimum decoder significantly outperforms the conventional decoder. The optimum decoder is more robust to weak source-relay link conditions.**

***Index Terms*—Relay Channels, cooperative, DF, coding, optimum decoding, error bound**

I. Introduction

Cooperative communication tries to exploit idle radio nodes in the vicinity of the sending or receiving nodes by establishing a proper transmission/reception strategy to further improve the system performance. One of the common strategies called decode and forward (DF) [1] is a form of regenerative relaying where the helping relay node decode, re-encode, and then forward the received message to the final destination. The destination node combines its observations on the directly transmitted signal and the forwarded signal to obtain a reliable decoding of the main message. As previously observed the DF scheme suffers from errors happened at the regenerative relay. Conventional decoding used for DF scheme ignores the relay error and can be easily miss-led by high reliability of the relay-destination link while the directly-received signal is far more reliable than the indirectly received one. Some remedies are already proposed to alleviate this shortcoming. Effective solutions could be i) selective relaying [1], where a relay only forwards when correctly receives data, ii) soft forwarding techniques [2]-[4], where the forwarded signal to some level maintains soft information on the reliability of the relay forwarded data, and iii) enhanced decoding techniques [5]-[6], where destination properly scales or clips the reliability information extracted from indirectly received signal and then combines it with directly received signal. Inspired by the attempts of the third category here we try to answer the question of what is the final performance limit and what is the optimal method to be used at the final decoder if a persistent regenerative relaying is exercised. Here optimal decoding is proposed to be based on a derived likelihood function. Computation of this function requires marginalization of so called partially observed likelihood function over all possible relay decoder outcomes. This means that the optimal decoding should be performed over the joint source-relay code space. For the class of binary-input channels Bhattacharyya bound is used to approximate the relay error. This has the benefit of allowing full factorization of the partially observed likelihood function and thus an affordable implementation of the derived optimal decoder. Tight error performance bound is derived and it is shown that calculation of the bound requires introduction of efficient weight analyzer tools over the joint source-relay code space.

The rest of the paper is organized as follows. The system model is presented in Section II. Section III derives the optimum decoding of the cooperative DF scheme. Analytical error bound for binary-input channels are derived in section IV. Section V provides numerical and simulation results and section VI concludes the paper.

II. System Model

Let's assume a simple cooperative communication system composed of three nodes: source node S, relay node R, and destination node D. Let's further assume that nodes transmission-reception is based on a simple protocol composed of two phases. In the first phase of this protocol S broadcasts its signal to R and D, and in the second phase only R transmits to D. Even though more efficient approach is to allow S and R jointly transmit in the second phase, for the convenience of introduction of the proposed approach we adhere to this simple protocol. The Cooperative communication model using this protocol is depicted in Figure 1. The links S-R, S-D, and R-D are enumerated with 0, 1, and 2, respectively. Let's define the symbol channels $\left(\mathcal{X}_j, p_{\boldsymbol{\theta}_j}\left(\boldsymbol{y}_j \middle| \boldsymbol{x}_j\right), \Theta_j, \mathcal{Y}_j\right)$, for j=0,1,2, by input alphabet $\mathcal{X}_j$, output alphabet $\mathcal{Y}_j$, and channel state space Θ_j, channel transition probability $p_{\boldsymbol{\theta}_j}\left(\boldsymbol{y}_j \middle| \boldsymbol{x}_j\right)$ $\boldsymbol{x}_j \in \mathcal{X}_j$, $\boldsymbol{y}_j \in \mathcal{Y}_j$, and $\boldsymbol{\theta}_j \in \Theta_j$. Apparently for the considered setup $\mathcal{X}_0 = \mathcal{X}_1$.

Phases I and II are composed of n_1, and n_2 transmissions, respectively.

Definition 1: A $\left(2^{n\bar{R}}, n_1, n_2\right)$ cooperative full decode and forward code with overall rate $\bar{R}$ and n=n_1+n_2 consists of the following:

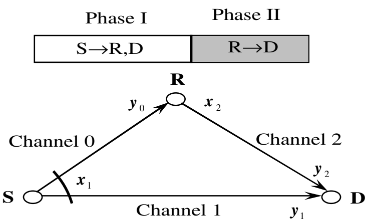

Figure 1 A Cooperative communication model composed of a source, destination, and single relay.

- A message set $\mathcal{W} = \left(1,\ldots,2^{n\bar{R}}\right)$;
- A source encoder $\mathcal{C}_1 : \mathcal{W} \rightarrow \mathcal{X}_1^{n_1}$ that maps the message $w_1 \in \mathcal{W}$ to the signal sequence $\underline{\boldsymbol{x}}_1(w_1) = \left(\boldsymbol{x}_{1,1}(w_1),\ldots,\boldsymbol{x}_{1,n_1}(w_1)\right) \in \mathcal{X}_1^{n_1}$;
- A relay encoder $\mathcal{C}_2 : \mathcal{W} \rightarrow \mathcal{X}_2^{n_2}$, that maps the message $w_2 \in \mathcal{W}$ to the signal sequence $\underline{\boldsymbol{x}}_2(w_2) = \left(\boldsymbol{x}_{2,1}(w_2),\ldots,\boldsymbol{x}_{2,n_2}(w_2)\right) \in \mathcal{X}_2^{n_2}$;
- A relay decoder $\mathcal{D}_0 : \mathcal{Y}_0^{n_1} \rightarrow \mathcal{W}$, that maps the received signal sequence $\underline{\boldsymbol{y}}_0 = \left(\boldsymbol{y}_{0,1},\ldots,\boldsymbol{y}_{0,n_1}\right) \in \mathcal{Y}_0^{n_1}$ to the decoded message $w_2 \in \mathcal{W}$;
- A destination decoder $\mathcal{D}_{12} : \mathcal{Y}_1^{n_1} \times \mathcal{Y}_2^{n_2} \rightarrow \mathcal{W}$, that maps the received sequences $\underline{\boldsymbol{y}}_1 = \left(\boldsymbol{y}_{1,1},\ldots,\boldsymbol{y}_{1,n_1}\right) \in \mathcal{Y}_1^{n_1}$ and $\underline{\boldsymbol{y}}_2 = \left(\boldsymbol{y}_{2,1},\ldots,\boldsymbol{y}_{2,n_2}\right) \in \mathcal{Y}_2^{n_2}$ to the decoded message $\hat{w} \in \mathcal{W}$.

Coherent reception assumption: It is assumed that the channel state sequences are perfectly known to the corresponding receivers. Therefore it is implicitly assumed that the state sequence $\underline{\boldsymbol{\theta}}_0 = \left(\boldsymbol{\theta}_{0,1},\ldots,\boldsymbol{\theta}_{0,n_1}\right) \in \Theta_0^{n_1}$ is available to $\mathcal{D}_0$ and the state sequences $\underline{\boldsymbol{\theta}}_1 = \left(\boldsymbol{\theta}_{1,1},\ldots,\boldsymbol{\theta}_{1,n_1}\right) \in \Theta_1^{n_1}$ and $\underline{\boldsymbol{\theta}}_2 = \left(\boldsymbol{\theta}_{2,1},\ldots,\boldsymbol{\theta}_{2,n_2}\right) \in \Theta_2^{n_2}$ are available to $\mathcal{D}_{12}$.

III. Optimum Decoding

The optimal decoding for $\mathcal{D}_{12}$ is to maximize the likelihood of the received sequences $\underline{\boldsymbol{y}}_1$, and $\underline{\boldsymbol{y}}_2$:

$$\hat{w}^{ML} = \arg\max_{w_1} P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2 \middle| w_1\right), \tag{1}$$

A decoder that maximizes the likelihood of its received signals is called maximum likelihood (ML) decoder. The likelihood $P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2 \middle| w_1\right)$ can be computed by marginalization of $P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, w_2 \middle| w_1\right)$ over all possible decoded w_2 messages:

$$P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2 \middle| w_1\right) = \sum_{w_2} P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, w_2 \middle| w_1\right), \tag{2}$$

Let's call $P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, w_2 \middle| w_1\right)$ as partially observed likelihood function, in the sense that only the transmitter part of the relay node (w_2) is observable.

Considering the orthogonal transmission of the two phases:

$$\begin{aligned} &P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, w_2 \middle| w_1\right) = \\ &p_{\underline{\boldsymbol{\theta}}_1}\left(\underline{\boldsymbol{y}}_1 \middle| \underline{\boldsymbol{x}}_1(w_1)\right) p_{\underline{\boldsymbol{\theta}}_2}\left(\underline{\boldsymbol{y}}_2 \middle| \underline{\boldsymbol{x}}_2(w_2)\right) P\left(w_2 \middle| w_1\right) = \\ &\prod_{j=1}^{2}\prod_{k=1}^{n_j} p_{\boldsymbol{\theta}_{j,k}}\left(\boldsymbol{y}_{j,k} \middle| \boldsymbol{x}_{j,k}(w_j)\right) P\left(w_2 \middle| w_1\right). \end{aligned} \tag{3}$$

$P\left(w_2 \middle| w_1\right)$ is the pairwise error probability of $\mathcal{D}_0$, i.e. probability of decoding w_2 while w_1 is transmitted.

Error free S-R link: In this case the pairwise error probability will be:

$$P\left(w_2 \middle| w_1\right) = \begin{cases} 1, & w_2 = w_1 \\ 0, & otherwise \end{cases}, \tag{4}$$

The error free S-R link assumption leads to the conventional decoding method for DF scheme:

$$\hat{w}_1^{DF} = \arg\max_{w_1} p_{\underline{\boldsymbol{\theta}}_1}\left(\underline{\boldsymbol{y}}_1 \middle| \underline{\boldsymbol{x}}_1(w_1)\right) p_{\underline{\boldsymbol{\theta}}_2}\left(\underline{\boldsymbol{y}}_2 \middle| \underline{\boldsymbol{x}}_2(w_1)\right), \tag{5}$$

However this assumption is not always valid. Let's assume that $\mathcal{D}_0$ is also ML, then the term $P\left(w_2 \middle| w_1\right)$ can be expressed as follows:

$$P\left(w_2 \middle| w_1\right) = \Pr\left\{ P\left(\underline{\boldsymbol{y}}_0 \middle| \underline{\boldsymbol{x}}_1(w_2)\right) \geq P\left(\underline{\boldsymbol{y}}_0 \middle| \underline{\boldsymbol{x}}_1(w_1)\right) \middle| w_1 \right\}, \tag{6}$$

A. Binary Input Channel

Let's assume the input alphabets $\mathcal{X}_1 = \mathcal{X}_2 = \{-1,+1\}$. In this case the $P\left(w_2 \middle| w_1\right)$ pairwise error probability can be approximated by Bhattacharyya bound [8]:

$$P\left(w_2 \middle| w_1\right) \leq B_0^{d_H\left(\underline{\boldsymbol{x}}_1(w_1), \underline{\boldsymbol{x}}_1(w_2)\right)}, \tag{7}$$

where $B_0 = E_{\boldsymbol{x}_0, \boldsymbol{y}_0, \boldsymbol{\theta}_0} \sqrt{\dfrac{p_{\boldsymbol{\theta}_0}\left(\boldsymbol{y}_0 \middle| -\boldsymbol{x}_0\right)}{p_{\boldsymbol{\theta}_0}\left(\boldsymbol{y}_0 \middle| \boldsymbol{x}_0\right)}}$

$d_H\left(\underline{\boldsymbol{x}}_1(w_1), \underline{\boldsymbol{x}}_1(w_2)\right)$ in above equation denotes Hamming distance between code sequences $\underline{\boldsymbol{x}}_1(w_1)$ and $\underline{\boldsymbol{x}}_1(w_2)$, which is the number of different components between the two sequences.

Substituting Bhattacharyya bound (7), in the expression (3) will result in:

$$\begin{aligned} &P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, w_2 \middle| w_1\right) \approx \\ &\prod_{j=1}^{2}\prod_{k=1}^{n_j} p_{\boldsymbol{\theta}_{j,k}}\left(\boldsymbol{y}_{j,k} \middle| \boldsymbol{x}_{j,k}(w_j)\right) B_0^{d_H\left(\underline{\boldsymbol{x}}_1(w_1), \underline{\boldsymbol{x}}_1(w_2)\right)} \\ &\propto \exp\left\{ \frac{1}{2}\sum_{j=1}^{2}\sum_{k=1}^{n_j} \lambda_{j,k} \boldsymbol{x}_{j,k}(w_j) + \frac{1}{2}\sum_{k=1}^{n_1} \mu_0 \boldsymbol{e}_{1k}(w_1, w_2) \right\}, \end{aligned}$$

$$\text{where } \mu_0 = -\log B_0,\ \boldsymbol{e}_{1k}(w_1, w_2) = \boldsymbol{x}_{1,k}(w_1)\boldsymbol{x}_{2,k}(w_2) \tag{8}$$

$\boldsymbol{e}_{1k}(w_1, w_2)$ is the k-th component of the error sequence at relay node where value of -1 indicates error at position k. In the above equation $\lambda_{j,k}$ are the bit reliabilities and are computed as follows:

$$\lambda_{j,k} = \log \frac{p_{\theta_{j,k}}\left(\boldsymbol{y}_{j,k} \middle| +1\right)}{p_{\theta_{j,k}}\left(\boldsymbol{y}_{j,k} \middle| -1\right)}, \text{ for } j=1,2, \text{ and } k=1,\ldots,n_j \tag{9}$$

The factorization obtained in (8) is quite useful and can be efficiently exploited to carry out the marginalization proposed in (2). For example marginalization can be performed over a graph jointly describing the codes $\mathcal{C}_1$ and $\mathcal{C}_2$. In this regard incorporation of Bhattacharyya bound is quite necessary as it allows complete factorization of the partially observed likelihood function.

B. Example Uncoded BPSK over SISO Fading Channel

Let's assume n_1=n_2=1. When all the three constituent channels are single-input single-output fading channels, their input-output relation will be expressed as:

$$\boldsymbol{y}_j = \boldsymbol{h}_j \sqrt{\gamma_j} \boldsymbol{x}_j + \boldsymbol{v}_j, \text{ for } j=0,1,2 \tag{10}$$

where $\boldsymbol{v}_j$ is additive zero-mean unit power circularly-symmetric white Gaussian noise. The bit reliability will be written as follows:

$$\lambda_j = 4\gamma_j \left|\boldsymbol{h}_j\right|^2 \operatorname{Re}\left\{\overline{\boldsymbol{y}}_j\right\}, \text{ for } j=0,1,2, \tag{11}$$

where $\overline{\boldsymbol{y}}_j = \boldsymbol{y}_j / \boldsymbol{h}_j \sqrt{\gamma_j}$ is the equalized output of channel j. The relay decoder will be reduced to a simple slicer: $\boldsymbol{x}_2 = sign(\lambda_0)$. The likelihood function $P\left(\boldsymbol{y}_1, \boldsymbol{y}_2 \middle| \boldsymbol{x}_1\right)$ will be:

$$P\left(\boldsymbol{y}_1, \boldsymbol{y}_2 \middle| \boldsymbol{x}_1\right) = e^{\frac{1}{2}\lambda_1 \boldsymbol{x}_1} \left((1-q) e^{\frac{1}{2}\lambda_2 \boldsymbol{x}_1} + q e^{-\frac{1}{2}\lambda_2 \boldsymbol{x}_1} \right)$$

$$\propto e^{\frac{1}{2}\lambda_1 \boldsymbol{x}_1} \left(e^{\frac{1}{2}\left(\lambda_2 \boldsymbol{x}_1 + L_q\right)} + e^{-\frac{1}{2}\left(\lambda_2 \boldsymbol{x}_1 + L_q\right)} \right) \tag{12}$$

In above equation q is the bit error probability at relay decoder $\mathcal{D}_0$ and is $q = Q\left(|\boldsymbol{h}_0|\sqrt{2\gamma_0}\right)$ if h_0 is perfectly known at final decoder $\mathcal{D}_{12}$ and is $q = E_{\boldsymbol{h}_0} Q\left(|\boldsymbol{h}_0|\sqrt{2\gamma_0}\right) \approx 1/2(1+\gamma_0)$ otherwise. $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-t^2/2} dt$ is the Gaussian Q function. $L_q = \log(1-q)/q$ is the SR link reliability. Let's define the overall bit reliability λ as:

$$\lambda \overset{\Delta}{=} \log \frac{P\left(\boldsymbol{y}_1, \boldsymbol{y}_2 \middle| \boldsymbol{x}_1 = +1\right)}{P\left(\boldsymbol{y}_1, \boldsymbol{y}_2 \middle| \boldsymbol{x}_1 = -1\right)} = \lambda_1 + \log \left(\frac{e^{\frac{1}{2}\left(L_q + \lambda_2\right)} + e^{-\frac{1}{2}\left(L_q + \lambda_2\right)}}{e^{\frac{1}{2}\left(L_q - \lambda_2\right)} + e^{-\frac{1}{2}\left(L_q - \lambda_2\right)}} \right)$$

$$\approx \lambda_1 + Clip\left(\lambda_2; L_q\right). \tag{13}$$

For the conventional DF the decoding error at R is ignored and thus $\lambda = \lambda_1 + \lambda_2$ is used as final bit LLR. Final decoded data will be $\hat{\boldsymbol{x}}_1 = sign(\lambda)$. Figure 2 depicts the decision boundaries of the conventional DF, clipped DF (CDF) and the optimum DF (ODF) by setting λ=0, for different SR link reliabilities L_q. Also shown is the joint constellation of S and R transmissions. Due to decoding error at R, the S and R transmissions encompass 4 possible combinations. Probability of each point's transmission is also shown in the figure. The points (-1,+1) and (+1,-1) are shadow points and start appearing as errors incur at relay decoder $\mathcal{D}_0$.

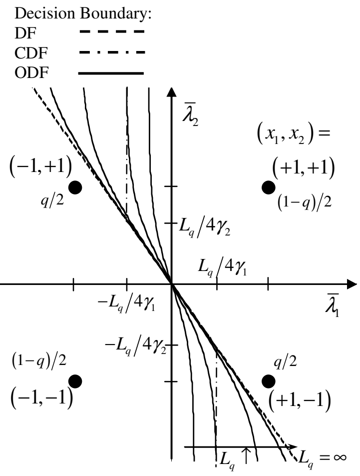

Figure 2 Source-relay hyper constellation and decision regions of DF, CDF, and ODF for binary uncoded cooperative system

IV. BER ANALYSIS

In this section the BER analysis is provided for the cooperative DF system with optimum decoding and binary input channels. Derivation of the performance bounds for non-binary input can take a similar approach. Let $P\left(w_1 \to \hat{w}_1 \middle| w_1\right)$ denotes the pairwise error probability (PEP) as the probability that the decoder $\mathcal{D}_{12}$ prefers $\hat{w}_1$ to the transmitted message w_1. Considering the optimum decoding criterion (1) PEP can be expressed as follows:

$$\begin{aligned}
&P\left(w_1 \to \hat{w}_1 \middle| w_1\right) \\
&= \Pr\left\{ P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2 \middle| \hat{w}_1\right) \geq P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2 \middle| w_1\right) \right\} \\
&\overset{(a)}{=} E_{w_2} \Pr\left\{ \begin{array}{l} \sum_{\hat{w}_2} P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, \hat{w}_2 \middle| \hat{w}_1\right) \\ \quad \geq \sum_{\tilde{w}_2} P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, \tilde{w}_2 \middle| w_1\right) \middle| w_2 \end{array} \right\} \\
&\overset{(b)}{\approx} E_{w_2} \Pr\left\{ \begin{array}{l} \max_{\hat{w}_2} P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, \hat{w}_2 \middle| \hat{w}_1\right) \\ \quad \geq \max_{\tilde{w}_2} P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, \tilde{w}_2 \middle| w_1\right) \middle| w_2 \end{array} \right\} \\
&\overset{(c)}{\leq} E_{w_2} \sum_{\hat{w}_2} \Pr\left\{ \begin{array}{l} P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, \hat{w}_2 \middle| \hat{w}_1\right) \\ \quad \geq P\left(\underline{\boldsymbol{y}}_1, \underline{\boldsymbol{y}}_2, w_2 \middle| w_1\right) \end{array} \right\} \\
&= \sum_{w_2, \hat{w}_2} P\left(w_2 \middle| w_1\right) P\left(\left(w_1, w_2\right) \to \left(\hat{w}_1, \hat{w}_2\right) \middle| w_1, w_2\right)
\end{aligned} \tag{14}$$

where (a) is obtained by substituting (2) and then conditioning and averaging with respect to relay decoded message w_2; (b) is obtained by approximating marginalization function with maximization; (c) is obtained first by using the obvious relation $\max_{\tilde{w}_2} P\left(\underline{\boldsymbol{y}}_1,\underline{\boldsymbol{y}}_2,\tilde{w}_2 \middle| w_1\right) \geq P\left(\underline{\boldsymbol{y}}_1,\underline{\boldsymbol{y}}_2,w_2 \middle| w_1\right)$, and then by applying the union bound over all the messages $\hat{w}_2 \in \mathcal{W}$ with respect to term $\max_{\hat{w}_2} P\left(\underline{\boldsymbol{y}}_1,\underline{\boldsymbol{y}}_2,\hat{w}_2 \middle| \hat{w}_1\right)$. The term

$$P\left((w_1,w_2)\rightarrow(\hat{w}_1,\hat{w}_2)\middle|w_1,w_2\right)=$$
$$\Pr\left\{P\left(\underline{\boldsymbol{y}}_1,\underline{\boldsymbol{y}}_2,\hat{w}_2\middle|\hat{w}_1\right)\geq P\left(\underline{\boldsymbol{y}}_1,\underline{\boldsymbol{y}}_2,w_2\middle|w_1\right)\right\}$$

represents joint-message pairwise error probability JM-PEP where the joint message $(\hat{w}_1,\hat{w}_2)$ is preferred to (w_1,w_2). The metric difference between the two joint messages is defined as:

$$\Delta=\log P\left(\underline{\boldsymbol{y}}_1,\underline{\boldsymbol{y}}_2,w_2\middle|w_1\right)-\log P\left(\underline{\boldsymbol{y}}_1,\underline{\boldsymbol{y}}_2,\hat{w}_2\middle|\hat{w}_1\right) \quad (15)$$
$$=\delta_0+\Delta_1+\Delta_2$$

where

$$\Delta_j=\log P\left(\underline{\boldsymbol{y}}_j\middle|w_j\right)-\log P\left(\underline{\boldsymbol{y}}_j\middle|\hat{w}_j\right) \text{ for } j=1,2$$

and $\delta_0=\log P\left(w_2\middle|w_1\right)-\log P\left(\hat{w}_2\middle|\hat{w}_1\right)$

Let $\boldsymbol{d}=(d_1,d_2)$ denote the Hamming distance distribution over channels j=1,2 with $d_j=d_H\left(\underline{\boldsymbol{x}}_j(w_j),\underline{\boldsymbol{x}}_j(\hat{w}_j)\right)$. Let also $\boldsymbol{\zeta}=\left(\zeta,\hat{\zeta}\right)$ with $\zeta=d_H\left(\underline{\boldsymbol{x}}_1(w_1),\underline{\boldsymbol{x}}_1(w_2)\right)$ and $\hat{\zeta}=d_H\left(\underline{\boldsymbol{x}}_1(\hat{w}_1),\underline{\boldsymbol{x}}_1(\hat{w}_2)\right)$. Using the Laplace transform of the probability density function (pdf) of Δ to calculate $P(\Delta\leq 0)$ [7], and after some derivations we can write:

$$f(\boldsymbol{d},\boldsymbol{\zeta})=P\left(w_2\middle|w_1\right)P\left((w_1,w_2)\rightarrow(\hat{w}_1,\hat{w}_2)\middle|w_1,w_2\right)$$
$$\leq\left(\frac{1}{2\pi i}\int_{\tilde{\alpha}-j\infty}^{\tilde{\alpha}+j\infty}\Phi_0^{\zeta}(s)\frac{ds}{s}\right)\left(\frac{1}{2\pi i}\int_{\alpha-j\infty}^{\alpha+j\infty}\Phi_{\boldsymbol{d},\boldsymbol{\zeta}}(s)\frac{ds}{s}\right)=f_{ub}(\boldsymbol{d},\boldsymbol{\zeta})$$

$$\text{where } \Phi_{\boldsymbol{d},\boldsymbol{\zeta}}(s)=\Phi_1^{d_1}(s).\Phi_2^{d_2}(s).e^{-s\left(\zeta-\hat{\zeta}\right)\mu_0} \quad (16)$$

Please note in the above that the same techniques is used to accurately calculate $P\left(w_2\middle|w_1\right)$. The parameters α and $\tilde{\alpha}$ belong to the intersection of the region of the convergence of the corresponding Laplace functions with the real positive line, respectively. Above integrals can be calculated by a method based on Gauss-Chebyshev quadrature rules [9]. $\Phi_j(s)=E_{\boldsymbol{x},\Delta|j}\left[e^{-s\Delta(\boldsymbol{x},-\boldsymbol{x})}\right]$, with $\Delta(\boldsymbol{x},\hat{\boldsymbol{x}})=\log p_{\boldsymbol{\theta}_j}\left(\boldsymbol{y}_j\middle|\boldsymbol{x}\right)-\log p_{\boldsymbol{\theta}_j}\left(\boldsymbol{y}_j\middle|\hat{\boldsymbol{x}}\right)$, for j=0,1,2 are derived based on binary-input assumption and using scrambling to symmetrize the channels. Calculation of $\Phi_j(s)$, j=0,1,2 depends on the channel characteristics and can be analytically calculated for most of the typical channels. Due to the enforced symmetry in the constituent channels the PEP $P\left(w_1\rightarrow\hat{w}_1\middle|w_1\right)$ will only depend on the Hamming distance d_1. The final word error probability will also be independent of the sent message w_1. Using union bound the word error probability will be:

$$P_\varepsilon=\sum_{\hat{w}_1}P\left(w_1\rightarrow\hat{w}_1\middle|w_1\right)\leq\sum_{\mathcal{E}}f_{ub}\left(\boldsymbol{d}_{\mathcal{E}},\boldsymbol{\zeta}_{\mathcal{E}}\right), \quad (17)$$
$$=\sum_{\boldsymbol{d},\boldsymbol{\zeta}}a(\boldsymbol{d},\boldsymbol{\zeta})f_{ub}(\boldsymbol{d},\boldsymbol{\zeta})$$

where $\mathcal{E}$ denotes any joint message error event $\left(0,w_2(\mathcal{E})\right)\rightarrow\left(\hat{w}_1(\mathcal{E}),\hat{w}_2(\mathcal{E})\right)$ given the message $w_1=0$ is sent. $\boldsymbol{d}_{\mathcal{E}}$ and $\boldsymbol{\zeta}_{\mathcal{E}}$ denote the respective Hamming distance distributions of the error event $\mathcal{E}$. The first summation is over all possible error events. In the second summation $a(\boldsymbol{d},\boldsymbol{\zeta})$ denotes the number of error events with Hamming distance distribution $(\boldsymbol{d},\boldsymbol{\zeta})$. Similarly bit error probability P_b can be upper bounded by:

$$P_b\leq\frac{1}{K_c}\sum_{\boldsymbol{d},\boldsymbol{\zeta}}a_I(\boldsymbol{d},\boldsymbol{\zeta})f_{ub}(\boldsymbol{d},\boldsymbol{\zeta}) \quad (18)$$

where $a_I(\boldsymbol{d},\boldsymbol{\zeta})$ denotes the total input weight of the error events with Hamming distance distribution $(\boldsymbol{d},\boldsymbol{\zeta})$. $K_c=n\bar{R}$ is the number of information bits per message. Coefficients $a(\boldsymbol{d},\boldsymbol{\zeta})$ and $a_I(\boldsymbol{d},\boldsymbol{\zeta})$ describe the Hamming weight structure of the joint code space $\mathcal{C}_1\times\mathcal{C}_2$ and efficient tools will be required to calculate these coefficients for the given codes $\mathcal{C}_1$ and $\mathcal{C}_2$.

V. NUMERICAL AND SIMULATION RESULTS

Here we provide some simulation results to demonstrate the efficiency of the proposed optimum decoding of DF scheme (ODF) as well as to show the tightness of the derived bit error rate (BER) bound. As explained in the previous sections optimum decoding requires an efficient decoding algorithm to do the marginalization proposed in (2) over the joint code space $\mathcal{C}_1\times\mathcal{C}_2$. Also calculation of the derived bounds (17) and (18) requires effective tools to calculate weight structure of the joint code space.

Derivation of the efficient decoding algorithms and weight analysis tools is not in the scope of this paper, hence here we provide the simulation and analysis results for a simple (7,4) Hamming codes. This code will allow implement the proposed optimal decoder and compute the derived bounds. Figure 4 shows the simulated BER performance and the derived bounds for both conventional decoder, denoted by DF, and optimal decoder, denoted by ODF, under fast Rayleigh fading conditions for all the constituent channels. Due to the limitation of space the derivation of the performance bound for conventional decoder is not presented here. The constituent channels' SNRs are adjusted relative to the channel 1 (S-D) SNR: $\gamma_0=\Delta_{sr}.\gamma_1$, $\gamma_2=\Delta_{rd}.\gamma_1$, where Δ_{sr} and Δ_{rd} represent SNR offsets of these two channels. As it is observed both the derived bounds are tight, and the ODF significantly outperforms DF. ODF has a performance gain around 4 dB at BER 10^{-3} with respect to the conventional decoder. Also shown in this figure is the performance of a so called clipped DF (CDF), an enhanced version of DF scheme [5] that clips the bit reliabilities (13) of the indirect signal before combining with directly received ones and then uses conventional decoding.

The considered SNR configuration in Figure 4 is a bad scenario for DF scheme as the S-R channel condition is worse than the condition of the R-D channel. This causes the signal received at phase II wrongly dominates the performance. Figure 4 depicts the BER upper bound surfaces for a fixed S-D SNR value of 5 dB and different values of Δ_{sr} and Δ_{rd} SNR offsets. As it is observed ODF has increased robustness of the system to S-R SNR. The BER performance of direct transmission is based on repetition coding, i.e. S repeats its transmission in the second phase. This figure provides insight on what SNR configurations cooperative transmission will be useful. ODF requires around Δ_{sr}=5 dB to overtake the direct transmission performance, while requires 3 dB more to perform better than direct transmission. DF and ODF require same value of $\Delta_{rd}\geq 0$ to become effective.

VI. Conclusion

In this paper optimum decoding of cooperative full decode and forward scheme is discussed and a likelihood function that properly takes relay decoding error into account is proposed. The proposed likelihood function is computed by marginalization of a so called partially observed likelihood function over all possible outcomes of the relay decoder. Then binary-input channels are further treated and using Bhattacharyya bound on the relay decoding error a full factorization of the partially observed likelihood function is obtained. This structure can be utilized to compute the likelihood function by an efficient marginalization over the joint source-relay code space. An efficient analytical tool is derived to calculate tight error bounds for the derived optimal decoder. Calculation of the derived bounds requires efficient tools to compute the weight structure of the joint source-relay code.

VII. Acknowledgment

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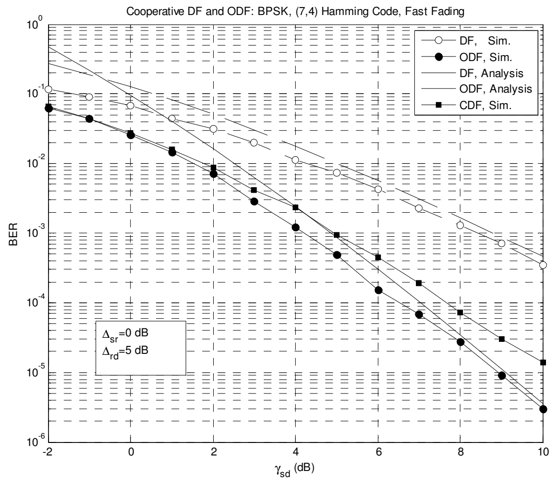

Figure 4, BER performance of a cooperative full decode and forward scheme under fast fading condition with BPSK modulation and Hamming (7,4) code and with conventional decoding DF, and optimum decoding ODF for different link SNR offsets.

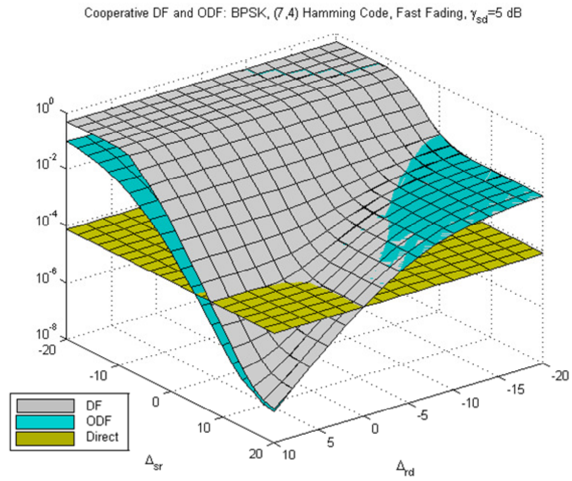

Figure 4, BER upper bound surfaces for a cooperative full decode and forward scheme under fast fading condition with BPSK modulation and Hamming (7,4) code. Shown are the performances for conventional decoding DF, optimal decoding ODF, and also included is direct transmission performance.

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