Future Network and Mobile Summit 2010 Conference Proceedings
Paul Cunningham and Miriam Cunningham (Eds)
IIMC International Information Management Corporation, 2010
ISBN: 978-1-905824-16-8

Low-Complexity Power Allocation Schemes for the Downlink of Nonregenerative Cooperative Multi-User MIMO Communication System

Fabien Héliot, Reza Hoshyar, and Rahim Tafazolli
Centre for Communication Systems Research (CCSR), Faculty of Electronics & Physical Sciences, University of Surrey, Guildford GU2 7XH, UK
Tel: +44 1483689492, Fax: + 44 1483686011, Email:F.Heliot@surrey.ac.uk

Abstract: Amplify-and-forward (AF) is one of the most popular and simple approaches to transmit information over a cooperative multi-input multi-output (MIMO) relay channel. In this paper, we propose two novel power allocation methods for the downlink of multi-user multi-input multi-output AF cooperative system, which are designed to optimise the sum-rate of the cooperative system according to the weighted sum-rate criterion. The main optimisation problem is not concave and our two methods aim at simplifying it in order to turn it into a concave problem, which can then be easily solved.

Keywords: Cooperative communication, amplify-and-forward, multi-input multi-output, multi-user, downlink

1. Introduction

Cooperative communication has recently attracted considerable research interests [1–6]. Cooperative communication uses one or several relays to improve the coverage and enhance the spectral efficiency of wireless communication. Relay node (RN) can be either used in a regenerative, i.e., decode and forward (DF), or in a non-regenerative way, i.e., amplify-and-forward (AF). In DF, the full decoding of the source message followed by the forwarding of the whole message to the destination node (DN) via the RN are performed. Whereas in AF, the RN amplifies and forwards the signal received from the source node (SN).

In a single-user multi-input multi-output (MIMO) cooperative scenario, the RN was first used as a simple equal gain amplifier, i.e., original AF scheme [6]. However, it has recently been shown in [7, 8] that it can also be utilised as a smart precoder by using a precoding matrix to fine-tune the power allocation over the relay channel, and thus, improve the spectral efficiency of the cooperative system. In the multi-user (MU) case, some methods have recently been proposed to efficiently design the precoding matrix at the RN but for the cases where users have a single antenna only [9, 10]. In this paper, we develop two power allocation methods for the downlink (DL) of MU MIMO nonregenerative cooperative system where all the nodes of the system have multiple antennas. Moreover, our methods are based on the weighted sum-rate (WSR) criterion, instead of the sum-rate criterion, as it is the case in [9, 10]. Note that the sum-rate criterion is a special case of the WSR. In order to design our precoding matrix, we assume as in [7,8] that the transmit signal covariance matrix and the RN to destination node (DN) link channel state information (CSI) are known at the RN.

Our novel power allocation method is designed according to the DL cooperative MIMO system model, which is introduced in Section 2. In Section 3, we explain the conditions for which the power allocation problem under WSR and total power constraint can be concave. Then, we introduce two methods to simplify the problem and turn it into a concave problem, which can be easily solved by a low-complexity algorithm that is provided. One of the method is based on the RN-SN aggregate channel block diagonalisation and the second method is based on its QR decomposition. In Section 4, the sum-rate performance of both our methods are illustrated and it turns out that the second method provides better performance than the first one. The result also show that a proper choice of weights can increase the sum-rate. Finally, conclusions are drawn in Section 5.

2. DL MU MIMO Cooperative System Model

We consider a cooperative MU MIMO communication system that is composed of $L+2$ nodes, where a SN, which is equipped with n antennas, cooperates with a nonregenerative RN, which is equipped with q antennas, to transmit data to L DNs, which are equipped with r_l antennas, as it is depicted in Fig. 1. In addition, we define $r = \sum_{l=1}^{L} r_l$.

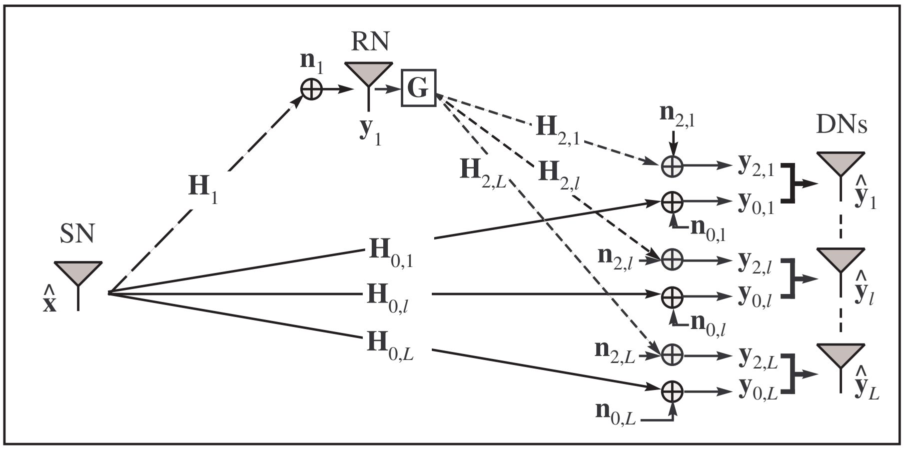

Figure 1: Nonregenerative cooperative DL MU MIMO communication system model.

For the simplicity of the introduction, we assume a half duplex relaying scenario with two equal duration phases as in [7,8], where in the first phase the SN broadcast the signal $\widehat{\mathbf{x}} = \sum_{l=1}^{L} \mathbf{x}_l$ to the DNs and RN, and in the second phase only the RN transmits to the DNs. The transmit signal is received by each DN as $\mathbf{y}_{0,l} = \mathbf{H}_{0,l}\widehat{\mathbf{x}} + \mathbf{n}_{0,l}$ and by the RN as $\mathbf{y}_1 = \mathbf{H}_1\widehat{\mathbf{x}} + \mathbf{n}_1$ at the end of the first phase, where $\mathbf{H}_{0,l} \in \mathbb{C}^{r_l \times n}$ as well as $\mathbf{H}_1 \in \mathbb{C}^{q \times n}$ characterise the MIMO channel of each SN-DN link and the SN-RN link, respectively. During the second phase, the signal $\mathbf{y}_1$ is amplified by using the precoding matrix $\mathbf{G} \in \mathbb{C}^{q \times q}$, then is transmitted towards the DNs and is received as $\mathbf{y}_{2,l} = \mathbf{H}_{2,l}\mathbf{G}\mathbf{y}_1 + \mathbf{n}_{2,l}$ by each DN, where $\mathbf{H}_{2,l} \in \mathbb{C}^{r_l \times q}$ characterises the MIMO channel of each RN-DN link. Moreover, each of the channel matrices $\mathbf{H}_{0,l}$, $\mathbf{H}_1$ $\mathbf{H}_{2,l}$ is a random matrix having i.i.d. complex Gaussian entries with zero-mean and unit variance. Furthermore $\mathbf{n}_{0,l} \in \mathbb{C}^{r_l \times 1}$, $\mathbf{n}_1 \in \mathbb{C}^{q \times 1}$ and $\mathbf{n}_{2,l} \in \mathbb{C}^{r_l \times 1}$ are vectors of independent zero-mean complex Gaussian noise entries with a variance of σ^2. The system model of the DL MU MIMO cooperative communication system introduced in Fig. 1 can be summarised as follows

$$\widehat{\mathbf{y}} = \begin{bmatrix} \mathbf{y}_0 \\ \mathbf{y}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_0 \\ \mathbf{H}_2\mathbf{G}\mathbf{H}_1 \end{bmatrix} \widehat{\mathbf{x}} + \begin{bmatrix} \mathbf{I}_r & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2\mathbf{G} & \mathbf{I}_r \end{bmatrix} \begin{bmatrix} \mathbf{n}_0 \\ \mathbf{n}_1 \\ \mathbf{n}_2 \end{bmatrix}, \tag{1}$$

where $\widehat{\mathbf{y}} = [\widehat{\mathbf{y}}_1^\dagger, \widehat{\mathbf{y}}_2^\dagger, \ldots, \widehat{\mathbf{y}}_L^\dagger]^\dagger \in \mathbb{C}^{2r\times 1}$, $\mathbf{y}_i = [\mathbf{y}_{i,1}^\dagger, \mathbf{y}_{i,2}^\dagger, \ldots, \mathbf{y}_{i,L}^\dagger]^\dagger \in \mathbb{C}^{r\times 1}$, $\mathbf{n}_i = [\mathbf{n}_{i,1}^\dagger, \mathbf{n}_{i,2}^\dagger, \ldots, \mathbf{n}_{i,L}^\dagger]^\dagger \in \mathbb{C}^{r\times 1}$, $\mathbf{H}_2 = [\mathbf{H}_{2,1}^\dagger, \mathbf{H}_{2,2}^\dagger, \ldots, \mathbf{H}_{2,L}^\dagger]^\dagger \in \mathbb{C}^{r\times q}$, $\mathbf{H}_0 = [\mathbf{H}_{0,1}, \mathbf{H}_{0,2}, \ldots, \mathbf{H}_{0,L}] \in \mathbb{C}^{r\times n}$, $\mathbf{I}_r$ is a $r \times r$ identity matrix, and $(.)^\dagger$ denotes the conjugate transpose operator. The cooperative mutual information of each user can then be expressed as [11]

$$I(\widehat{\mathbf{y}}_l; \mathbf{x}_l) = \frac{1}{2}\log_2 \left| \mathbf{I}_{2r_l} + \mathbf{H}_l \mathbf{R}_{\mathbf{x}_l} \mathbf{H}_l^\dagger \mathbf{R}_{\mathbf{n},l}^{-1} \right| = \frac{1}{2}\log_2 \begin{vmatrix} \mathbf{A}_l & \mathbf{D}_l \\ \mathbf{C}_l & \mathbf{B}_l \end{vmatrix}, \tag{2}$$

where

$$\mathbf{H}_l = \begin{bmatrix} \mathbf{H}_{0,l} \\ \mathbf{H}_{2,l}\mathbf{G}\mathbf{H}_1 \end{bmatrix}, \mathbf{R}_{\mathbf{n},l} = \begin{bmatrix} \mathbf{R}_{\mathbf{n}_{0,l}} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{2,l}\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,l}}\mathbf{G}^\dagger\mathbf{H}_{2,l}^\dagger + \sigma^2\mathbf{I}_{r_l} \end{bmatrix},$$

the factor 1/2 accounts for the two-phase transmission, $\mathbf{A}_l$, $\mathbf{B}_l$, $\mathbf{C}_l$ and $\mathbf{D}_l$ are matrices, $\mathbf{R}_{\mathbf{x}_l} = \mathrm{E}\left\{\mathbf{x}_l\mathbf{x}_l^\dagger\right\}$ is the l-th independent transmit message covariance matrix, $\mathrm{E}\{.\}$ stands for the expectation, $\mathbf{R}_{\mathbf{n}_{0,l}} = \sigma^2\mathbf{I}_{r_l} + \mathbf{H}_{0,l}\left(\sum_{j=1,j\neq l}^{L}\mathbf{R}_{\mathbf{x},j}\right)\mathbf{H}_{0,l}^\dagger$ and $\mathbf{R}_{\mathbf{n}_{1,l}} = \sigma^2\mathbf{I}_q + \mathbf{H}_1\left(\sum_{j=1,j\neq l}^{L}\mathbf{R}_{\mathbf{x},j}\right)\mathbf{H}_1^\dagger$ are noise plus residual interference covariance matrices. Dirty paper coding (DPC) [12] can be used at the SN to mitigate part of the interference, however, the full CSI of the relay channel must be known at the SN in order to apply it. The interference from the i-th user towards the l can be considered negligible for $i > l$ by applying DPC to user L first and then to the following descending order users. Consequently, $\mathbf{R}_{\mathbf{n}_{0,l}} = \sigma^2\mathbf{I}_{r_l} + \mathbf{H}_{0,l}\left(\sum_{i=1,}^{l-1}\mathbf{R}_{\mathbf{x}_i}\right)\mathbf{H}_{0,l}^\dagger$ and $\mathbf{R}_{\mathbf{n}_{1,l}} = \sigma^2\mathbf{I}_q + \mathbf{H}_1\left(\sum_{i=1,}^{l-1}\mathbf{R}_{\mathbf{x}_i}\right)\mathbf{H}_1^\dagger$ if DPC is applied at the SN. The direct link and relay link mutual information, i.e., $I(\mathbf{y}_{0,l}; \mathbf{x}_l)$ and $I(\mathbf{y}_{2,l}; \mathbf{x}_l)$, can also be computed by employing (2) for $\mathbf{H}_l = \mathbf{H}_{0,l}$, $\mathbf{R}_{\mathbf{n},l} = \mathbf{R}_{\mathbf{n}_{0,l}}$ and $\mathbf{H}_l = \mathbf{H}_{2,l}\mathbf{G}\mathbf{H}_1$, $\mathbf{R}_{\mathbf{n},l} = \mathbf{H}_{2,l}\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,l}}\mathbf{G}^\dagger\mathbf{H}_{2,l}^\dagger + \sigma^2\mathbf{I}_{r_l}$, respectively, such that

$$\begin{aligned} I(\mathbf{y}_{0,l}; \mathbf{x}_l) &= \frac{1}{2}\log_2|\mathbf{A}_l| = \frac{1}{2}\log_2\left|\mathbf{I}_{r_l} + \mathbf{H}_{0,l}\mathbf{R}_{\mathbf{x}_l}\mathbf{H}_{0,l}^\dagger\mathbf{R}_{\mathbf{n}_{0,l}}^{-1}\right| \\ I(\mathbf{y}_{2,l}; \mathbf{x}_l) &= \frac{1}{2}\log_2|\mathbf{B}_l| = \frac{1}{2}\log_2\left|\mathbf{I}_{r_l} + \mathbf{H}_{2,l}\mathbf{G}\mathbf{R}_l\mathbf{G}^\dagger\mathbf{H}_{2,l}^\dagger(\mathbf{H}_{2,l}\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,l}}\mathbf{G}^\dagger\mathbf{H}_{2,l}^\dagger + \sigma^2\mathbf{I}_{r_l})^{-1}\right|, \end{aligned} \tag{3}$$

where $\mathbf{R}_l = \mathbf{H}_1\mathbf{R}_{\mathbf{x}_l}\mathbf{H}_1^\dagger$. Moreover, $I(\widehat{\mathbf{y}}_l; \mathbf{x}_l)$ in (2) can be simplified and re-expressed as $I(\widehat{\mathbf{y}}_l; \mathbf{x}_l) =$

$$I(\mathbf{y}_{0,l}; \mathbf{x}_l) + \frac{1}{2}\log_2\left|\mathbf{I}_{r_l} + \mathbf{H}_{2,l}\mathbf{G}\mathbf{H}_1\mathbf{R}_{\mathbf{x},l}^{\frac{1}{2}}\widehat{\mathbf{A}}_l^{-1}\mathbf{R}_{\mathbf{x},l}^{\frac{\dagger}{2}}\mathbf{H}_1^\dagger\mathbf{G}^\dagger\mathbf{H}_{2,l}^\dagger(\mathbf{H}_{2,l}\mathbf{G}\mathbf{R}_{\mathbf{n}_{1,l}}\mathbf{G}^\dagger\mathbf{H}_{2,l}^\dagger + \sigma^2\mathbf{I}_{r_l})^{-1}\right| \tag{4}$$

by using [13], and where $\widehat{\mathbf{A}}_l = \mathbf{I}_n + \mathbf{R}_{\mathbf{x},l}^{\frac{\dagger}{2}}\mathbf{H}_{0,l}^\dagger\mathbf{R}_{\mathbf{n}_{0,l}}^{-1}\mathbf{H}_{0,l}\mathbf{R}_{\mathbf{x},l}^{\frac{1}{2}}$ is a positive definite matrix. Hence, $I(\widehat{\mathbf{y}}_l; \mathbf{x}_l) \leq I(\mathbf{y}_{0,l}; \mathbf{x}_l) + I(\mathbf{y}_{2,l}; \mathbf{x}_l)$ according to (3) and (4). Moreover, it can easily be proved that $I(\widehat{\mathbf{y}}_l; \mathbf{x}_l) \geq \min\{I(\mathbf{y}_{0,l}; \mathbf{x}_l), I(\mathbf{y}_{2,l}; \mathbf{x}_l)\}$, Thus, $I(\widehat{\mathbf{y}}_l; \mathbf{x}_l)$ can be increased by maximising $I(\mathbf{y}_{2,l}; \mathbf{x}_l)$, or equivalently by optimising $\mathbf{G}$ at the RN, as it has been recently shown in [8] for the single user case. In the following, we consider that $\sigma = 1$ and $\mathbf{R}_{\mathbf{x}_l} = \alpha_l \mathbf{I}_n$, where $\alpha_l = P_1/(nL)$ if each $\mathbf{x}_l$ signals have been allocated with the same power at the SN and P_1 is the total transmit power at the SN.

3. WSR based Power Allocation at the RN

The relay link mutual information that can be achieved by the weighted sum of the users is given according to (3) by

$$\Sigma_{\mathbf{y}_2,\mathrm{DL}} = \frac{1}{2} \sum_{l=1}^{L} w_l \log_2 |\mathbf{B}_l| \, , \tag{5}$$

where w_l is the l-th weight and $w_l \geq 0, \forall l \in [1, \ldots, L]$. The problem of maximising the weighted sum mutual information, or WSR, under the constraint that the transmit power at the RN should not exceed P_2 is such that

$$\max_{\mathbf{G}} \Sigma_{\mathbf{y}_2,\mathrm{DL}} \text{ s.t. } \mathbf{G} \geq 0; \mathrm{tr}\left(\mathbf{G}\mathbf{R}_{\mathbf{y}_1}\mathbf{G}^{\dagger}\right) \leq P_2, \tag{6}$$

where $\mathbf{R}_{\mathbf{y}_1} = \mathrm{E}\left\{\mathbf{y}_1\mathbf{y}_1^{\dagger}\right\} = \mathbf{I}_q + \mathbf{H}_1\left(\sum_{i=1,}^{L} \mathbf{R}_{\mathbf{x}_i}\right)\mathbf{H}_1^{\dagger}$. The function $\Sigma_{\mathbf{y}_2,\mathrm{DL}}$ would be concave if all the $\mathbf{B}_l$ are Hermitian positive definite matrices [14]. Therefore, in order to be able to solve this problem via classic convex/concave optimisation tools [14], i.e., interior point method, the problem becomes equivalent to find a $\mathbf{G}$ matrix subject that any $\mathbf{B}_l$ is an Hermitian positive definite matrix. Such a $\mathbf{G}$ matrix will be helpful to transform (6) into a concave optimisation problem but it would not ensure the optimality of $\mathbf{G}$. In the single user case, i.e., $l = 1$, it has been shown in [7, 8] that the optimal $\mathbf{G}$ matrix design is such that $\mathbf{G} = \mathbf{V}\widetilde{\mathbf{G}}\mathbf{U}_1^{\dagger}$ when assuming that the DN-RN and RN-DN link CSI $\mathbf{H}_1$ and $\mathbf{H}_{2,1}$, respectively, are known at the RN and where $\mathbf{V}$ is a column matrix that contains the q right-singular vectors of $\mathbf{H}_{2,1}$, $\widetilde{\mathbf{G}} = \mathrm{diag}(\sqrt{p_1}, \sqrt{p_2}, \ldots, \sqrt{p_q})$ is a $q \times q$ diagonal matrix and $\mathbf{U}_1$ is a column matrix that contains the q left-singular vectors of $\mathbf{H}_1$. As explained in [8], the Hadamard determinant theorem establishes that $I(\mathbf{y}_{2,1}; \mathbf{x_1}) = \frac{1}{2}\log_2 |\mathbf{B}_1|$ would be optimised when $\mathbf{B}_1$ is a diagonal matrix. Consequently, an optimal $\mathbf{G}$ matrix will be the one that turns any $\mathbf{B}_l$ into a diagonal matrix. However, such a $\mathbf{G}$ matrix may not always exist.

3.1 Aggregate Channel Block-Diagonalisation based Optimisation

We can straightforwardly derive for each user l an optimised $\mathbf{G}_l$ matrix, which diagonalised $\mathbf{B}_l$, by applying a similar approach as in [7, 8] with the assumption that $\mathbf{H}_1$ and every $\mathbf{H}_{2_l}$ are known at the RN; the CSI of each RN-DN link can be obtained by reciprocity. Let $\mathbf{H}_1\mathbf{H}_1^{\dagger}$ be decomposed via eigenvalue decomposition as $\mathbf{H}_1\mathbf{H}_1^{\dagger} = \mathbf{U}\widehat{\mathbf{\Lambda}}\mathbf{U}^{\dagger}$, where $\mathbf{U} \in \mathbb{C}^{q\times q}$ is a unitary matrix and $\widehat{\mathbf{\Lambda}}$ is a $q \times q$ diagonal matrix with diagonal elements $\widehat{\lambda}_i \in \mathbb{C}$, which are sorted in descending order [7, 8]. Then $\mathbf{R}_l = \mathbf{U}\mathbf{\Lambda}_l\mathbf{U}^{\dagger}$, is a $q \times q$ diagonal matrix with diagonal elements $\lambda_{l,i} = \alpha_l\widehat{\lambda}_i$. Moreover, let each $\mathbf{H}_{2,l}$ be decomposed via SVD as $\mathbf{H}_{2,l} = \mathbf{U}_{2,l}\widehat{\mathbf{\Omega}}_l^{\frac{1}{2}}\mathbf{W}_l^{\dagger}$, where $\mathbf{U}_{2,l} \in \mathbb{C}^{r_l\times r_l}$ and $\mathbf{W}_l \in \mathbb{C}^{q\times q}$ are unitary matrices, $\widehat{\mathbf{\Omega}}_l \in \mathbb{C}^{r_l\times q}$ is a rectangular diagonal matrices, and $\mathbf{\Omega}_l = \widehat{\mathbf{\Omega}}_l^{\frac{1}{2}}\widehat{\mathbf{\Omega}}_l^{\frac{1}{2}\dagger}$ is a $r_l \times r_l$ diagonal matrix with diagonal elements $\omega_{l,i} \in \mathbb{C}$, which are sorted in descending order as in [7, 8]. Consequently, we can diagonalise $\mathbf{B}_l$ and re-express $|\mathbf{B}_l|$ in (3) as

$$|\mathbf{B}_l| = \prod_{i=1}^{r_l}\left(1 + \frac{p_u\omega_u\alpha_l\widehat{\lambda}_u}{1 + p_u\omega_u(1 + \beta_l\widehat{\lambda}_u)}\right) \tag{7}$$

by applying $\mathbf{G}_l = \mathbf{W}_l\widetilde{\mathbf{G}}\mathbf{U}^\dagger$ in (3), where $u = i + \sum_{j=1}^{l-1} r_j$, $\beta_l = \sum_{i=1}^{l-1}\alpha_i$ or $\beta_l = \sum_{j=1,j\neq l}^{L}\alpha_j$ if either DPC is used or not, respectively, at the SN. However, we seek for a unique $\mathbf{G}$ matrix that diagonalise all the $\mathbf{B}_l$ and not a collection of $\mathbf{G}_l$ matrices that diagonalise $\mathbf{B}_l$ for each user only. In order to design such a $\mathbf{G}$ matrix, we can employ the same approach as in [15] to block-diagonalise the aggregate channel $\overline{\mathbf{H}}_2$ for the case of $r \le q$, and we can then define our $\mathbf{G}$ matrix as

$$\mathbf{G} = \widehat{\mathbf{V}}\widetilde{\mathbf{G}}\mathbf{U}^\dagger, \tag{8}$$

where $\widehat{\mathbf{V}} = [\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_L]$. Moreover, $\mathbf{V}_l$ is defined as $\mathbf{V}_l = \mathbf{V}_l^{(0)}\mathbf{V}_l^{(1)}$, where $\mathbf{V}_l^{(0)} = \mathbf{Y}_{l,[\rho_l^0+1:q]}$, $\mathbf{V}_l^{(0)} \in \mathbb{C}^{q\times(q-\rho_l^0)}$, $\mathbf{Y}_l$ is a matrix of rank ρ_l^0 that contains the q right-singular vectors of $\overline{\mathbf{H}}_{2,l} = [\mathbf{H}_{2,1}^\dagger, \dots, \mathbf{H}_{2,l-1}^\dagger, \mathbf{H}_{2,l+1}^\dagger, \dots, \mathbf{H}_{2,L}^\dagger]^\dagger$ and $\mathbf{Y}_{l,[\rho_l^0+1:q]}$ contains the last $q-\rho_l^0$ columns of $\mathbf{Y}_l$. Furthermore, $\mathbf{V}_l^{(1)} = \mathbf{Z}_{l,[1:r_l]}$, $\mathbf{V}_l^{(1)} \in \mathbb{C}^{(q-\rho_l^0)\times r_l}$, $\mathbf{Z}_l$ is a matrix that contains the $q-\rho_l^0$ right-singular vectors of $\mathbf{H}_{2,l}\mathbf{V}_l^{(0)}$ and $\mathbf{Z}_{l,[1:r_l]}$ contains the first r_l columns of $\mathbf{Z}_l$. Consequently, the problem in (6) simplifies as

$$\max_{\mathbf{p}_2} \Sigma_{\mathbf{y}_2,\mathrm{DL}} = \frac{1}{2}\sum_{l=1}^{L} w_l \log_2|\mathbf{B}_l| \text{ s.t. } \mathbf{p_2} \ge 0; \sum_{l=1}^{L}\sum_{i=1}^{r_l} p_u(1+(P_1/n)\widehat{\lambda}_u) \le P_2, \tag{9}$$

by applying (8) in (6) and where $|\mathbf{B}_l|$ is given in (7). This problem can be easily solved with the following low-complexity algorithm with ω_u is the i-th nonnegative eigenvalues of $\mathbf{H}_{2,l}\mathbf{V}_l^{(0)}\left(\mathbf{V}_l^{(0)}\right)^\dagger\mathbf{H}_{2,l}^\dagger$. Note that this technique can be extended for the case where $r > q$ by resorting to eigenmode selection per user or user selection [15].

Algorithm 1

1: ***Input:*** $w_l, a_l, \beta_l, \forall l \in [1, L]$, $\omega_u, \widehat{\lambda}_u, \forall u \in [1, r]$, ϵ;
2: Set $x = [\max_u\{x_u\}]_+$, where $x_u = \frac{1+(P_1/n)\widehat{\lambda}_u}{w_l\omega_u(a_l-\beta_l)\widehat{\lambda}_u}$;
3: Solve $f(x) = \sum_{l=1}^{\widehat{L}}\sum_{i=1}^{\widehat{r}_l}\widehat{p}_u(x)(1+(P_1/n)\widehat{\lambda}_u) < P_2 + \epsilon, x \ge 0$ by using the Newton-Raphson method [16] and obtain a solution $x^\star$.
4: Use $x^\star$ to compute $p_u = \widehat{p}_u(x^\star), \forall u \in [1, q]$.
5: ***Output:*** $\mathbf{G} = \widehat{\mathbf{V}}\,\mathrm{diag}(\sqrt{p_1}, \sqrt{p_2}, \dots, \sqrt{p_q})\mathbf{U}^\dagger$.

In the algorithm above, $[.]_+ = \max\{., 0\}$, $\sum_{l=1}^{\widehat{L}}\widehat{r}_l = r$ or q if either $r \le q$ or $r > q$, respectively, $\widehat{p}_u(x) =$

$$\left[\frac{-(2+(\alpha_l+2\beta_l)\widehat{\lambda}_u) + \sqrt{\alpha_l\widehat{\lambda}_u\left(\alpha_l\widehat{\lambda}_u + \frac{4w_l\omega_u(1+(\alpha_l+\beta_l)\widehat{\lambda}_u)(1+\beta_l\widehat{\lambda}_u)x}{1+(P_1/n)\widehat{\lambda}_u}\right)}}{2\omega_u(1+(\alpha_l+\beta_l)\widehat{\lambda}_u)(1+\beta_l\widehat{\lambda}_u)}\right]_+ .$$

3.2 QR Decomposition based Optimisation

It has recently been shown in [10] for the single antenna per user case that $\mathbf{B}_l$ can be diagonalised by applying the QR decomposition to the aggregate RN-DN channel $\mathbf{H}_2$ at the RN. The QR decomposition transform $\mathbf{H}_2$ into a triangular matrix and, consequently, turns the $\mathbf{B}_l$ matrix into a diagonal matrix. Let us now assume that each user has several antennas, $\mathbf{H}_2^\dagger$ can be decomposed as $\mathbf{H}_2^\dagger = \widehat{\mathbf{V}}\mathbf{S}$, where $\widehat{\mathbf{V}} \in \mathbb{C}^{q\times q}$ is a unitary matrix and $\mathbf{S} \in \mathbb{C}^{q\times r}$ is a upper triangular matrix. Then, we can express $\mathbf{H}_2$ as $\mathbf{H}_2 = \mathbf{S}^\dagger\widehat{\mathbf{V}}^\dagger$ and this will incur some simplifications in (6), however, it will not turn $\mathbf{B}_l$ into a Hermitian positive definite matrix and (6) will not be concave, which is contrary to the single antenna per user case. In order to simplify the formulation of the problem in (6) and ensure that $\mathbf{B}_l$ is Hermitian positive definite, we assume that the elements $s_{i,j}, j > i$ of $\mathbf{S}$ are negligible and instead of finding the $\mathbf{G}$ matrix that solves (6) for $\mathbf{H}_2^\dagger = \widehat{\mathbf{V}}\mathbf{S}$, we aim at finding the $\mathbf{G}$ matrix that solves (6) for $\mathbf{H}_2^\dagger = \widehat{\mathbf{V}}\mathbf{T}$, where $\mathbf{T} = \mathrm{diag}(\mathbf{S})$. On the one hand, this approximation allows us to simplify further the problem in (6) such that it becomes equivalent to the problem in (9) and, hence, the $\mathbf{G}$ matrix can swiftly be obtained by using **Algorithm 1** but where $\omega_u = \|s_{i,i}\|^2$. On the other hand, this $\mathbf{G}$ matrix will be a sub-optimal solution of the original problem.

4. Results

Our novel AF power allocation methods, which have been introduced in Sections 3., are compared against each other in terms of sum-rate performance for either equal weights, or weights that are calculated according to the channel norm of each user. The user with the largest norm is the first user. Moreover, the ordering of the users has then been made according to their respective weights.

In our simulations, we denote $\mathrm{SNR}_1(\mathrm{dB}) = \log_{10}(P_1)$ and $\mathrm{SNR}_2(\mathrm{dB}) = \log_{10}(P_2)$. We consider an equal power allocation for each user, i.e., $\alpha_l = P_1/(nL)$, and that the direct links between the SN and DNs are weak in comparison with the relay links and, hence, can be neglected. A single-tap independent and identically distributed (i.i.d) Rayleigh fading channel is assumed between the various links, SN-RN, and RN-DNs. We considered 5×10^3 realisations of each channel for our sum-rate analysis. Note that the parameter ϵ which is used to fine-tune the accuracy of **Algorithm 1**, has been set to $\epsilon = 10^{-5}$.

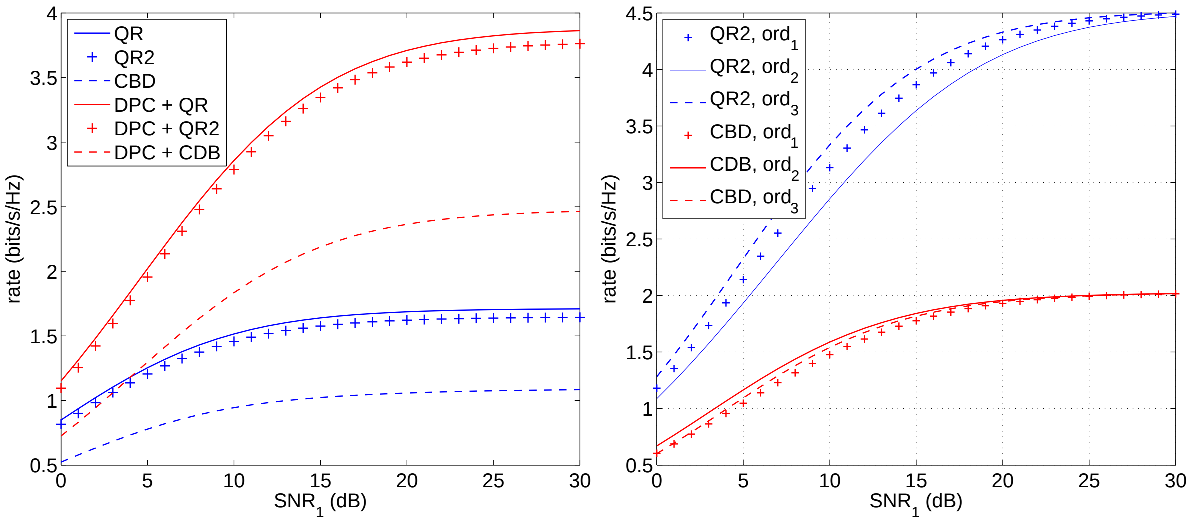

Figure 2: Sum-rate performance of various power allocation schemes (left) and eigenvalues ordering performance variation (right).

In the left-hand side of Fig. 2, we plot the sum-rate performance of the QR decomposition method ($\mathbf{H}_2^\dagger = \widehat{\mathbf{V}}\mathbf{S}$), which is denoted QR and has been obtained through exhaustive search, of our QR decomposition method with approximation $\mathbf{H}_2^\dagger = \widehat{\mathbf{V}}\mathbf{T}$, which is denoted QR2, and of the aggregate channel block-diagonalisation, which is denoted CBD. These curves have been obtained by setting $\text{SNR}_2 = 10$ dB, $L = 2$, $n = q = 4$, $w_l = 1, r_l = 2 \forall l \in [1, L]$ and by considering or not DPC at the SN. The results show that the QR method performs far better than the CBD method; it implies that the process of channel block-diagonalisation is not efficient. The result also indicate that our QR2 method performs only 2 to 5 % worst than the original QR method but with a far lower computational complexity. In the right-hand side of Fig. 2, we plot for $\text{SNR}_2 = 10$ dB, $L = 4$, $n = q = 8$, $w_l = 1$ and $r_l = 2$, $\forall l \in [1, L]$, the sum-rate performance of the QR2 and CBD methods for different ordering of the eigenvalues ω_u and $\widehat{\lambda}_u$. In the single user case, it has been pointed out in [7, 8] that a close to optimal ordering is obtained by pairing the best $\widehat{\lambda}_u$ with the best ω_u, then the second best $\widehat{\lambda}_u$ with the second best ω_u, and so on and so forth. Here we have considered three types of ordering, ord_1 correspond to the single user case; in ord_2, the best $\widehat{\lambda}_u$ is paired with the best ω_u of the first user, then the second best $\widehat{\lambda}_u$ is paired with the best ω_u of the second user, and so on and so forth; in ord_3, we sort the ω_u of each user in descending order and $\widehat{\lambda}_u$ as well. The results show the importance of the ordering of the eigenvalues and that the single user ordering is no longer optimal in the MU case.

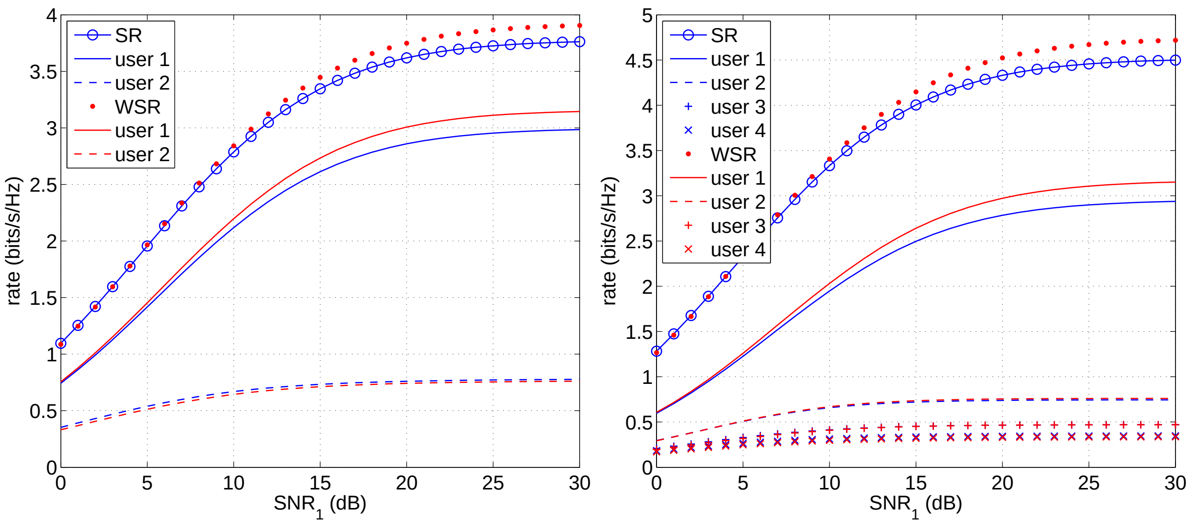

Figure 3: Cooperative sum-rate (blue) vs.weighted sum-rate (red) DL performance in function of SNR_1 (dB) for SNR_2=10 dB, $L = 2, n = q = 4, r_l = 2$ (left) and $L = 4, n = q = 8, r_l = 2$ (right).

In Fig. 3, we compare the sum of user rate performance of our QR2 decomposition method under sum-rate, i.e., $w_l = 1, \forall l \in [1, L]$, and WSR criteria, where the weights are computed according to the norm of $\mathbf{H}_{2,l}$ for each user. In the left-hand side of the figure, we consider a two-user case and in the right-hand side, a four-user case. The results show that by modifying the user weights, we can increases the overall performance. In both cases, by prioritising the first user, we can increases its rate without impairing the other user rates.

5. Conclusions

In this paper, we have introduced two novel power allocation methods for nonregenerative cooperative DL MU MIMO communication, which is designed to optimise the sum-rate of the cooperative system according to the WSR criterion. The main optimisation problem is not concave and our two methods aim at simplifying it in order to turn it into a concave problem. We have explained the conditions for which the power allocation problem under WSR and total power constraint can be concave. Then, we introduce two methods to simplify the problem and turn it into a concave problem, which can be easily solved by a low-complexity algorithm that is provided. The sum-rate performance of both our methods have presented and the method based on QR decomposition provides better performance than the other method. The result also show that a proper choice of weights can increase the sum-rate. Future work could be carried out by considering the joint power allocation at SN and RN, where the full CSI of the relay channel would be available at the SN, which is a prerequisite for the use of DPC.

Acknowledgment

This work has been performed in the framework of the FP7 project ROCKET IST-215282 STP, which is funded by the European Community.

References

[1] J. N. Laneman and G. W. Wornell, "Distributed Space-Time-Coded Protocols for Exploiting Cooperative Diversity in Wireless Networks," *IEEE Trans. Inform. Theory*, vol. 49, pp. 2415–2425, Oct. 2003.

[2] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity Part I - System Description," *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

[3] A. Sendonaris, E. Erkip, and B. Aazhang, "User Cooperation Diversity Part II - Implementation Aspects and Performance Analysis," *IEEE Trans. Commun.*, vol. 51, pp. 1939–1948, Nov. 2003.

[4] M. Janani, A. Heday, T. Hunter, and A. Nosratinia, "Coded Cooperation in Wireless Communications: Space-Time Transmission and Iterative Decoding," *IEEE Trans. Signal Processing*, vol. 52, pp. 362–371, Feb. 2004.

[5] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative Communicationin Wireless Networks," *IEEE Commun. Mag.*, vol. 42, pp. 74–80, Oct. 2004.

[6] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. Inform. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[7] Y. Fan and J. Thompson, "MIMO Configurations for Relay Channels: Theory and Practice," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1774–1786, May 2007.

[8] O. Muñoz-Medina, J. Vidal, and A. Agustn, "Linear Transceiver Design in Nonregenerative Relays with Channel State Information," *IEEE Trans. Signal Processing*, vol. 55, pp. 2593–2604, June 2007.

[9] L. Weng and R. Murch, "Multi-user MIMO Relay System with Self-Interference Cancelation," in *Proc. IEEE WCNC*, (Hong-Kong, China), Mar. 2007.

[10] C. Chae, T. Tang, R. Health, and S. Cho, "MIMO Relaying with Linear Processing for Multi-user Transmission in Fixed Relay Networks," *IEEE Trans. Inform. Theory*, vol. 56, pp. 727–738, Feb. 2008.

[11] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New-York, USA: ed. Wiley, 1991.

[12] M. H. M. Costa, "Writing on Dirty Paper," *IEEE Trans. Inform. Theory*, vol. 27, pp. 508–511, July 1981.

[13] "Imperial College Website, Block Matrix Determinant," ([Online]. Available: http://www.ee.ic.ac.uk/hp/staff/dmb/matrix/proof003.html).

[14] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, UK: Cambridge Univ. Press, 2004.

[15] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-Forcing Methods for Downlink Spatial Multiplexing in Multiuser MIMO Channels," *IEEE Trans. Signal Processing*, vol. 52, pp. 461–471, Feb. 2004.

[16] "Wolfram Mathworld Website."