

Improving Downlink Performance by Reusing the Subcarriers within the Cell

Muhammad Ali IMRAN, Reza HOSHYAR

Centre for Communication Systems Research, Guildford, GU2 7XH, UK

Tel: +44 1483 6002, Fax: + 44 1483 6011, Email:m.imran@surrey.ac.uk

Abstract: In this paper we consider the downlink of a single cell multicarrier system. For a group of uniformly distributed users, the problem of subcarrier allocation is studied when the constraint of orthogonal allocation of subcarriers is removed. Appropriate users are paired together and same subcarrier is simultaneously reused by the two users in the pair. One of the users, tolerates the presence of the other user as interference while the other user performs interference cancellation for the first user's signal before extracting the desired signal. This allows for a high power to be transmitted to the user with weak channel without degrading the performance of the subcarrier sharing user with a strong channel. Algorithms for appropriate pairing and subcarrier allocation are presented. Using a suitable mathematical model and some numerical calculations it is shown that the proposed scheme can provide system level gains in terms of information theoretic sum rate as compared to the exclusive subcarrier allocation.

Keywords: Downlink, Resource Allocation, Reusing, Overloading

1. Introduction

High data rate services for the cellular mobile systems are becoming increasingly popular. This requires an efficient use of the available spectrum while fairly serving all the system users. Higher data rate services are particularly in demand over the downlink of the cellular system due to the asymmetric nature of services like data download and web-browsing [1]. In a cellular system some users are in an advantageous situation when they are close to the base station of the cells. Other users, located at the boundary of the cells, are in a disadvantageous scenario. Over the downlink channel, in order to deliver an acceptable signal to noise ratio at the receiver of the cell-edge user, high signal power need to be transmitted from the base station. This results in a very strong interference to the users closer to the base station of the cell. To avoid this problem, the downlink scheduler aims to keep the transmissions orthogonal in time, frequency or code [2].

Consider the downlink of the orthogonal frequency division multiplexing (OFDM) system. Usually each subcarrier is allocated to a single user within each cell (termed as OFDMA). The subcarrier is re-used in another cell at a reasonable physical distance from the cell where it is being used, keeping the other cell (inter-cell) interference to a low value. This scheme performs well in terms of interference minimisation however each user is restrained to exclusively use specific portion of the spectrum.

The exclusive subcarrier allocation schemes are studied in detail (along with bit and power loading problems) in [3, 4]. However, the condition of exclusively using each subcarrier for only a single user, is maintained. Bergman and Cover in [5] introduced the novel idea of cooperative broadcasting where two users can be simultaneously served over the downlink channel if their channel gains are significantly diverse: one user

having very strong channel and the other having very weak channel. The main idea is that any signal transmitted to the user with the weak channel will be more likely to be decodable by the user with the strong channel. Hence the user with the strong channel can perform interference cancellation by first decoding the interference signal and then re-modulating and subtracting it from the aggregate received signal to recover its desired signal. The user with the weak channel will tolerate the interference caused by the signals transmitted to the other user with strong channel (due to strong channel the corresponding signal is relatively weaker in power and causes less interference).

In this paper we compare the performance of a single cell downlink system for the two different subcarrier allocation schemes. The first scheme uses exclusive subcarrier allocation. The second scheme pairs the users with diverse channel gains and for every pair the two users simultaneously use the same channel, employing the cooperative broadcasting approach. The constrained power for the sub-carrier is shared between the two users in a specific ratio. We use the term subcarrier reusing (or overloading) to describe this process of allocating the two users, the same subcarrier and the allocation scheme is termed as the subcarrier reusing scheme. The two schemes are compared for the information theoretic sum rate and the benefit of user pairing scheme are shown.

The rest of the paper is organised as follows. The next section describes the system model used and how the sum rate is calculated for each of the allocation scheme. Section 3 describes the algorithms used to carry out the exclusive allocation and the subcarrier reuse allocation scheme. Section 4 presents some results for the sum rate of the two schemes and section 5 concludes the paper.

2. System Model

Consider the downlink of a single cell, serving K users (indexed by k) with M orthogonal subcarriers (indexed by m). The set of all user indices is given by \mathcal{K} and the set of all subcarrier indices is given by \mathcal{M} . We consider the problem of allocation of these subcarriers to the users for improving the sum-rate of the system with the constraint that each user is allocated at least one subcarrier. Total transmit power constraint for each subcarrier is P Watts at the base station transmitter.

Each user is randomly located within the cellular coverage extent. For the wireless communication environment, we represent the magnitude squared channel gain from the base station to a user terminal k , for a subcarrier m , by $h_{k,m}$. For all users and subcarriers, these channel gains are composed of two factors. One factor ($\psi_{k,m}$) models the distance dependent path loss experienced by the signal and the other factor ($g_{k,m}$) models the Rayleigh fading. We assume a narrow-band, single-tap flat (frequency non-selective), Rayleigh fading channel for each subcarrier and for this work we ignore the log-normal shadowing effects. The (magnitude squared) channel gain can be expressed in terms of these two factors as: $h_{k,m} = \psi_k \times g_{k,m}$ where ψ_k is the distance dependent power loss (same for all subcarriers seen by the user k).

The distance dependent path loss factor is modelled using the inverse power law:

$$\psi_k = L_0 \left(\frac{D_0}{\hat{d}_k} \right)^\eta \quad (1)$$

where the constant L_0 defines the loss at the reference distance, \hat{d}_k is the physical distance between the source (base station antenna) and the destination (user terminal)

antenna) and η is the path loss exponent which determines the rate at which the received signal power decays as the separation between the source and destination is increased. For system based calculations a slight modification to the path loss model is required to avoid the situation where the separation between the source and destination is less than the reference distance D_0 . The modified path loss model is given as [6]:

$$\psi_k = L_0 \left(\frac{D_0}{D_0 + d_k} \right)^\eta \quad (2)$$

where $d_k = \hat{d}_k - D_0$ is the distance measured between the user terminal and a point at the reference distance from the base station. This ensures that the separation between the source and the destination is not less than unity. This simplified model fits a wide range of empirical path loss models [7] obtained for various environments by appropriately choosing the parameters L_0 and η . Factor $g_{k,m}$ is the magnitude squared value of a Rayleigh fading gain and hence modelled as an exponentially distributed random variable with mean normalised to 1.

As a specific case (extension to more general case is straightforward), we assume the same number of subcarriers as the user: $|\mathcal{M}| = |\mathcal{K}|$. We consider two types of allocations.

2.1 Exclusive Allocation

The exclusive allocation is where one subcarrier is assigned to only one user. Let $\mathcal{A}_m = \{k_m\}$ where $k_m \in \mathcal{K}$, represents the index of the user using subcarrier m . Each subcarrier is used by exactly one user: $|\mathcal{A}_m| = 1 \forall m$. All users are assigned at least one subcarrier: $\bigcup_{m=1}^M \mathcal{A}_m = \mathcal{K}$. The signal to noise and interference ratio for each subcarrier m is given as:

$$\text{SNIR}_m^A = \frac{h_{k_m,m}P}{\sigma^2} \quad (3)$$

where k_m is the index of the user using subcarrier m and σ^2 is the noise power. The information theoretic sum-rate in exclusive subcarrier allocation scenario is given as:

$$R_A = \sum_{m=1}^M \log_2 \left(1 + \text{SNIR}_m^A \right) = \sum_{m=1}^M \log_2 \left(1 + \frac{h_{k_m,m}P}{\sigma^2} \right) \quad (4)$$

2.2 Subcarrier Reuse Allocation

In subcarrier reuse allocation, one carrier is shared between two users. As above, the allocation set $\mathcal{A}_m = \{l_m, s_m\}$ gives the indices of the two users sharing the subcarrier m . For the pair of users using subcarrier m , we identify the index of the dominant user l_m and the index of the weak user s_m using the check: $h_{l_m,m} \geq h_{s_m,m}$. User l_m is given αP power-share and the user s_m gets $(1 - \alpha)P$. For a subcarrier m , the signal to noise and interference ratio for the two users l_m, s_m is given by:

$$\text{SNIR}_{s_m}^B = \frac{h_{s_m,m}(1 - \alpha)P}{h_{s_m,m}\alpha P + \sigma^2} \quad (5)$$

$$\text{SNIR}_{l_m}^B = \frac{h_{l_m,m}\alpha P}{\sigma^2} \quad (6)$$

Sum rate for all the subcarriers is given as:

$$R_B = \sum_{m=1}^M [\log_2 (1 + \text{SNIR}_{s_m}^B) + \log_2 (1 + \text{SNIR}_{l_m}^B)] \quad (7)$$

$$= \sum_{m=1}^M \left[\log_2 \left(1 + \frac{h_{s_m,m}(1-\alpha)P}{h_{s_m,m}\alpha P + \sigma^2} \right) + \log_2 \left(1 + \frac{h_{l_m,m}\alpha P}{\sigma^2} \right) \right] \quad (8)$$

To perform the allocations mentioned above (with the objective of maximising the R_A and R_B), we propose the algorithms described in the next section.

3. Pairing and Subcarrier Allocation

For the exclusive subcarrier allocation, a simple greedy algorithm (Algorithm 1) is used which is described below. Note that the justification for using a greedy algorithm as the benchmark, comes from the fact that our objective is the sum-rate maximisation for the system.

Algorithm 1 Greedy Exclusive Subcarrier Allocation

Require: Set of user indices \mathcal{K} and the subcarrier indices \mathcal{M}

- 1: $\mathcal{M}_c = \mathcal{M}, \mathcal{K}_c = \mathcal{K}, \mathcal{H}_c = \{h_{k,m} | m \in \mathcal{M}, k \in \mathcal{K}\}$
 - 2: **for** M iterations **do**
 - 3: $h_{k,m} \leftarrow \max(\mathcal{H}_c)$
 - 4: $\mathcal{A}_m \leftarrow \{k\}$
 - 5: $\mathcal{M}_c = \mathcal{M}_c \setminus m, \mathcal{K}_c = \mathcal{K}_c \setminus k$
 - 6: $\mathcal{H}_c = \{h_{k,m} | m \in \mathcal{M}_c, k \in \mathcal{K}_c\}$
 - 7: **end for**
-

The algorithm starts (Line 1) with the evolving sets of the available subcarriers (\mathcal{M}_c) and the unassigned users (\mathcal{K}) initialised to contain all users and all subcarriers. A set of all channel gains is also initialised as \mathcal{H}_c . A loop is run (Line 2) as many times as the number of subcarriers that need to be loaded : M in our case. Using a greedy allocation policy, the channel gain with highest magnitude is selected (Line 3) and the corresponding user k and the subcarrier m are identified. The identified user index is assigned to the bin (Line 4) and the subcarrier index and the user index are removed from the evolving sets of available subcarriers and unassigned users (Line 5). The set of available channel gains is updated by removing all channels corresponding to the assigned user and the subcarrier (contains the channel gains for only the users and subcarriers that are still unassigned). The algorithm returns M single element sets $\mathcal{A}_m = \{k_m\}$ defining the user indices k_m that are assigned to use each subcarrier m .

We use the output of Algorithm 1 and optimally (over-)load another user on each subcarrier using Algorithm 2. This algorithm is presented below. For each subcarrier m taken one-by-one (Line 1) we identify the set of users that can be overloaded (for the reuse of the subcarrier) other than the already allocated user (Line 2). For each possible option of the paired user k , a pairing \mathcal{A}_c is formed (Line) and the sum rate is evaluated using (8) for the pairing (Line 4). The pairing option k that maximises the sum-rate is selected (Line 5) and the pairing allocation set defined for the subcarrier m (Line 6).

In the next section the performance of the two algorithms is compared for the obtained sum-rate.

Algorithm 2 User Pairing for Subcarrier Reuse Allocation

Require: k_m for all m from Algorithm 1

- 1: **for** subcarrier $m = 1$ to $m = M$ **do**
 - 2: **for all** users $k \in \mathcal{A} \setminus k_m$ **do**
 - 3: $\mathcal{A}_c = \{k_m, k\}$
 - 4: Use (8) to find $R_B(k)$ for the allocation \mathcal{A}_c
 - 5: $k_n \leftarrow \max_k R_B(k)$
 - 6: $\mathcal{A}_m = \{k_m, k_n\}$
 - 7: **end for**
 - 8: **end for**
-

4. Results and Discussions

Using the mathematical model described and selecting the propagation parameters from [7] to simulate the distance dependent path loss, we investigate the performance of the proposed algorithm in the single cell scenario. We consider 10 users uniformly distributed over the span of the cell radius (1km for the simulation) and all 10 subcarriers are assumed to be fading independently. Since Rayleigh flat fading is assumed the magnitude squared channel gain is exponentially distributed [8].

Each cell has a total transmit power constraint of $P = 1\text{W}$ per subcarrier. The path gain at the reference distance of 1m is $L_0 = -38\text{dB}$ and the path loss exponent $\eta = 3.5$ is considered to correspond to empirical models. Noise spectral density $N_0 = -169\text{ dBm}$ is assumed and a carrier of narrow band spectrum ($W = 50\text{ KHz}$) is considered where the noise power is calculated as $\sigma^2 = N_0W$. The other cell interference is assumed to be 1mW (when it is held fixed). Several system snapshots (1000 for the results presented in this paper) are simulated and their results are averaged to show the mean sum-rate performance.

Algorithm 1 is used for the greedy subcarrier allocation and Algorithm 2 is used for the pairing and reusing of each subcarrier. For exclusive subcarrier allocation, each user transmits on a subcarrier with no contention with other users and hence experiences no interference. Information theoretic sum-rate is evaluated using (4). In case of reused subcarrier allocation, sum rate is evaluated for all the subcarriers using (8). On each subcarrier, the total power P is also shared between the two overloaded users (reusing the same subcarrier) with a specified ratio (fraction αP given to the user with the stronger channel and the fraction $(1 - \alpha)P$ given to the user with the weak channel).

In Fig. 1 we plot the sum-rate of the system as a function of varying the share of power (α) for the user with the stronger channel. Note that, this power sharing is only performed when two users are served simultaneously using the same subcarrier. Hence the sum rate for the exclusive allocation scheme is independent of power sharing factor α . When subcarrier reuse scheme is employed, the fraction α is selected based on the required fairness. It shall be noted that the smaller value of α gives more preference to the users with relatively weaker channels but this results in a sacrifice of the sum-rate of the system. As shown in Fig. 1, for the scenario described above, the values of α less than 0.7 reduce the sum-rate from the case of exclusive allocation of the subcarriers (the solid line). The values of α above 0.7 provide significant improvement in sum-rate capacity as compared to the exclusive allocation case. However this improvement comes at a cost of fairness between the two users sharing the same subcarrier. One reason for this improvement is that in the greedy allocation algorithm, as the allocations are

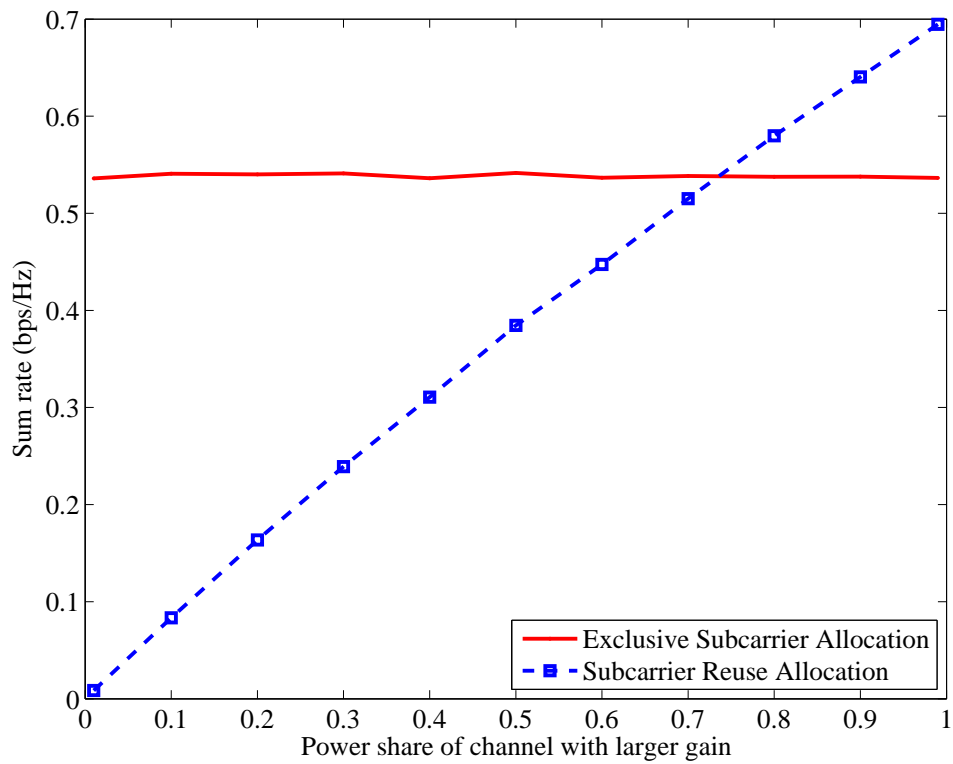


Figure 1: Sum rate of the system for varying the power share (α) of the channel with larger channel gain (strong channel)

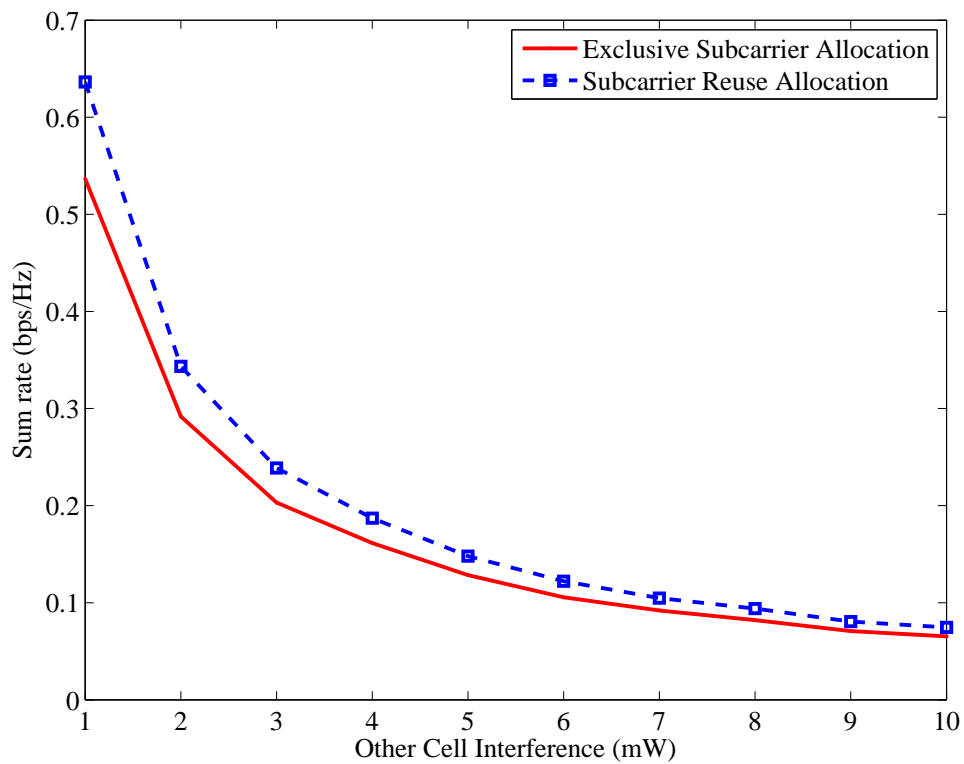


Figure 2: Effect of other cell interference on the sum rate of the system, $\alpha = 0.9$

made, the choices for the subcarriers that can be allocated to the remaining users, are more and more limited. As an example, the last subcarrier allocation has only one possible allocation. When the subcarrier reuse allocation algorithm is used following the first allocation phase, it has the capability of reusing some subcarriers with good channel gains which were not used in the exclusive allocation since they had already been allocated to some other user.

In Fig. 2 the sum rate performance is evaluated as a function of the other-cell interference. A power sharing factor $\alpha = 0.9$ is used. It can be observed that over a wide range of other-cell interference, the subcarrier reuse algorithm performs better in comparison to the exclusive allocation algorithm. However, the gain becomes smaller for the stronger other-cell interference. In practical systems, the other cell interference can be controlled by carefully allocating the subcarriers to the cells ensuring that the same subcarrier is used only in the cells that are physically well separated. Due to the distance dependent path loss, the other cell interference for the same subcarrier is minimised.

5. Conclusions and Future Directions

In this paper we have compared the downlink sum-rate of a multicarrier system when two different schemes are used for subcarrier allocation to the users. In one scheme each subcarrier is exclusively used by only one user at a time. In the other scheme each subcarrier can be further reused with one additional user sharing the same subcarrier by simultaneously using it along with the other user. For the two users loaded on the same subcarrier, the users with stronger and weaker channel are identified. For the user with the stronger channel, sum rate is evaluated assuming perfect interference cancellation at the receiver. On the other hand, for the user with the weaker channel the sum rate is calculated assuming that the interference is tolerated. The subcarrier reuse allocation scheme is shown to perform better in terms of the sum-rate specially when a larger share of power is given to the users with stronger channel in each pair. The gain reduces for stronger inter-cell interference. It shall be noted that the subcarrier reuse allocation scheme requires extra processing at the receiver of the user with stronger channel (for interference cancellation).

As a possible extension of this work we are modelling the correlated fading of the subcarriers and will also do the straightforward extension of the proposed algorithms, when the number of users and the number of subcarriers are not the same. We are also performing the analysis of the user-rate share (cumulative distribution for the user rates) as a function of varying the power share factor α . Consideration of a more realistic multicell scenario and the possibility of coordination and cooperation of the base stations is also an interesting direction for further extensions.

Acknowledgment

This work was performed as part of the ROCKET project which has received research funding from the EU FP7 framework.

References

- [1] A. J. Goldsmith, "The capacity of downlink fading channels with variable rate and power," *IEEE Transactions on Vehicular Technology*, vol. 46, pp. 569–580, 1997.

- [2] J. Andrews, "Interference cancellation for cellular systems: a contemporary overview," *Wireless Communications, IEEE*, vol. 12, pp. 19–29, April 2005.
- [3] Y. J. Zhang and K. Letaief, "Multiuser adaptive subcarrier-and-bit allocation with adaptive cell selection for OFDM systems," *Wireless Communications, IEEE Transactions on*, vol. 3, pp. 1566–1575, Sept. 2004.
- [4] C. Y. Wong, R. Cheng, K. Lataief, and R. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," *Selected Areas in Communications, IEEE Journal on*, vol. 17, pp. 1747–1758, Oct 1999.
- [5] P. P. Bergmans and T. M. Cover, "Cooperative broadcasting," *IEEE Transactions on Information Theory*, vol. IT-20, pp. 317–324, 1974.
- [6] S. Chatzinotas, M. A. Imran, and C. Tzaras, "On the capacity of variable density cellular systems under multicell decoding," *IEEE Communication Letters*, vol. 12, pp. 496–498, July 2008.
- [7] T. S. Rappaport, ed., *Wireless Communications: Principles and Practice*. Upper Saddle River, NJ, USA: Prentice-Hall, Inc., 1995.
- [8] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.