A CLOSED-FORM APPROXIMATION OF THE OUTAGE PROBABILITY FOR DISTRIBUTED MIMO SYSTEMS

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ABSTRACT

The instantaneous capacity of multiple-input multiple-output (MIMO) systems over Rayleigh fading channel is known to follow a normal probability distribution when the number of transmit and receive antennas is large. In this paper, we extend this result for distributed MIMO (DMIMO) systems by deriving a closed-form approximation of the outage probability which becomes tighter as the rate or the number of nodes and antennas of the system increases. This expression can be used to easily evaluate the outage probability of DMIMO systems, both for downlink and uplink cases. We also utilize this expression to compare the theoretical throughput performance of MIMO and DMIMO systems and show that DMIMO systems can achieve a throughput gain over their MIMO counterparts by taking advantage of spatial diversity.

1. INTRODUCTION

Calculation of the channel capacity for multiple-input multiple-output (MIMO) systems over Rayleigh fading channel has attracted considerable research interests in the past decades [1–3]. Recently, it has been shown by various authors using different methods that the mutual information, also known as instantaneous capacity, of the MIMO Rayleigh fading channel is equivalent to a Gaussian random variable (RV) [4–7]. This result has been theoretically obtained for a large number of inputs and outputs of the MIMO channel and has been confirmed by simulations for even a small number of them [5, 6]. Moreover, this result has been used in [7] to approximate the outage probability of MIMO systems.

In distributed MIMO (DMIMO) systems, several static nodes equipped with several antennas cooperate to communicate with a mobile node. This can be performed in a multi-hop fashion where one of the static node communicates with the mobile node using other static nodes to relay the information [8]. Alternatively, it can be achieved in a distributed manner where each static node shares the same data via a backhaul link and all the static nodes communicate together with the mobile node [9]. The latter scenario is considered here.

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In this paper, we propose a closed-form approximation of the outage probability for the downlink and uplink of DMIMO systems, relying on the system model introduced in Section 2. Applying a similar approach as in [4, 7] based on the replica method [10] and the multiple saddle point integration technique [11], it can be shown that the instantaneous capacity of DMIMO systems over Rayleigh fading channel is asymptotically equivalent to a Gaussian RV [12]. In the MIMO case [4, 7], the mean and variance of the equivalent Gaussian RV are given in terms of the roots of a degree-2 polynomial, and the uniqueness of the outage probability approximation depends on the uniqueness of these moments, which itself is related to the selection of one root amongst the two possible roots of the degree-2 polynomial. This selection is straightforward in the MIMO case. Similarly, the uniqueness of our closed-form approximation, which is proposed in Section 2, depends on the selection of one root amongst $m+1$ possible roots. However, in the DMIMO case, the choice of the root is a complex problem by itself which is first solved in Section 3 for the case of $m=2$, and it is then generalized for any other m values. In Section 4, we indicate that the accuracy of our approximation increases as the rate or the number of nodes and antennas of the system increases. Next, we utilize our expression to show the throughput gain that DMIMO systems can achieve over MIMO systems when an unlimited number of automatic retransmission requests (ARQs) is assumed. Finally, conclusions are drawn in Section 5.

2. DISTRIBUTED MIMO SYSTEM MODEL

We consider a DMIMO communication system composed of $m+1$ nodes which are in different locations, where m base stations (BSs) equipped of p antennas cooperate to transmit/receive data to/from a mobile station (MS) equipped with q antennas, as illustrated in Fig. 1. The matrices $\mathbf{\Sigma}_i$ and $\mathbf{H}_i$ characterize the average path loss/shadowing and the MIMO Rayleigh fading channel, respectively, between the i-th BS and the MS, $i \in \{1, m\}$. The equivalent channel model of the system depicted in Fig. 1 is then defined as $\widetilde{\mathbf{H}} = \mathbf{\Sigma} \odot \mathbf{H}$, where $\mathbf{\Sigma} = [\mathbf{\Sigma}_1^\dagger, \mathbf{\Sigma}_2^\dagger, \ldots, \mathbf{\Sigma}_m^\dagger]^\dagger$, $\mathbf{H} = [\mathbf{H}_1^\dagger, \mathbf{H}_2^\dagger, \ldots, \mathbf{H}_m^\dagger]^\dagger$, $(.)^\dagger$ is the Hermitian transpose operator and $\odot$ is the entry-wise product between any two matrices. Moreover, $\widetilde{\mathbf{H}} \in$

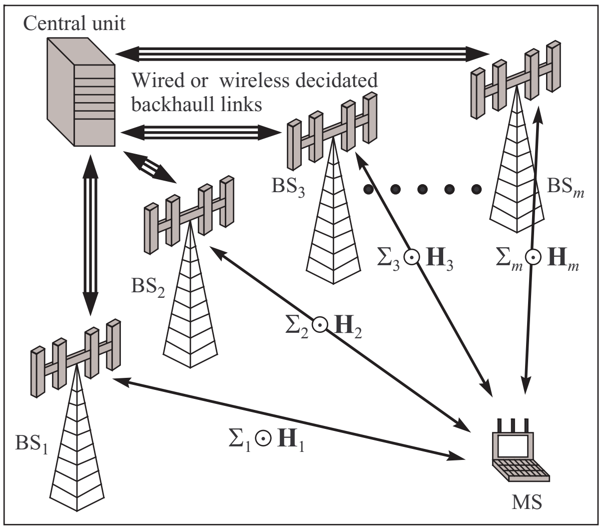

Fig. 1. Distributed MIMO system model

$\mathbb{C}^{N_r \times N_t}$, $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ and $\mathbf{\Sigma} \in \mathbb{R}_{+}^{N_r \times N_t}$, with $\mathbb{R}_+ = \{x \in \mathbb{R} | x \geq 0\}$. The total number of transmit and receive antennas of the DMIMO system is defined as N_t and N_r, respectively. In the downlink case $N_t = mp$, $N_r = q$ and $n = p$, whereas in the uplink case $N_t = n = q$ and $N_r = mp$, where n is the number of transmit antenna per node. Accordingly, the receive signal $\mathbf{r} \in \mathbb{C}^{N_r \times 1}$ can be expressed as

$$\mathbf{r} = \sqrt{\epsilon/n}\widetilde{\mathbf{H}}\mathbf{s} + \mathbf{n}, \tag{1}$$

where ϵ is the average transmitted signal energy, $\mathbf{s} \in \mathbb{C}^{N_t \times 1}$ is a normalized to unit-power transmitted signal and $\mathbf{n} \in \mathbb{C}^{N_r \times 1}$ is a vector of independent zero-mean complex Gaussian noise entries with a double-sided variance of $N_0/2$.

We consider that $\mathbf{H}$ is a random matrix having zero-mean and variance of 0.5 independently and identically distributed (i.i.d.) Gaussian entries. We also assume that once $\mathbf{H}$ has been chosen, it then remains fixed for the whole transmission. In this case, the channel is non-ergodic and the outage probability, i.e, the probability that the transmission rate R exceeds the instantaneous capacity of the channel $C(\widetilde{\mathbf{H}})$, is defined as

$$P_{\text{out}} \triangleq \mathrm{P}\left(C(\widetilde{\mathbf{H}}) < R\right), \tag{2}$$

where

$$C(\widetilde{\mathbf{H}}) \triangleq \lambda \ln \left| \mathbf{I}_{N_r} + \frac{\gamma}{n}\widetilde{\mathbf{H}}\widetilde{\mathbf{H}}^{\dagger} \right|, \tag{3}$$

$\gamma \triangleq \epsilon/N_0, \gamma \in \mathbb{R}_+$, is the average signal-to-noise-ratio (SNR), $\mathbf{I}_{N_r}$ is a $N_r \times N_r$ identity matrix, $\widetilde{\mathbf{H}}^{\dagger}$ indicates the Hermitian transpose of $\widetilde{\mathbf{H}}$, and $\lambda = 1$ if the capacity is expressed in nats/s/Hz or $\lambda = \frac{1}{\ln(2)}$ if the capacity is expressed in bits/s/Hz.

Recently, it has been shown in [4–7] that the instantaneous capacity $C(\mathbf{H})$ of MIMO systems over Rayleigh fading channel, i.e., for $m = 1$, tends to be a Gaussian RV as p and q grows to infinity. Following a similar approach as the one described in [4, 7] , we have shown in [12] that $C(\widetilde{\mathbf{H}})$ is asymptotically equivalent to a Gaussian RV for large values of p, q and $m > 1$, with its mean and variance are given by

$$\begin{cases} \mu_{C(\widetilde{\mathbf{H}})} &= -\lambda n \left[(m\alpha + \beta)\ln(\omega) + \alpha \sum_{i=1}^{m} \ln\left(\frac{\upsilon_i}{d_0 + \omega\upsilon_i}\right) \right. \\ & \quad \left. + \beta \ln\left(\frac{d_0}{\beta}\right) + \alpha \sum_{i=1}^{m} \frac{d_0}{d_0 + \omega\upsilon_i} \right], \\ \sigma^2_{C(\widetilde{\mathbf{H}})} &= -\lambda^2 \ln\left(1 - \frac{\alpha}{\beta}\sum_{i=1}^{m}\left(\frac{d_0}{d_0 + \omega\upsilon_i}\right)^2\right), \end{cases} \tag{4}$$

where $\alpha \triangleq p/n$, $\beta \triangleq q/n$, $\omega \triangleq 1/\sqrt{\gamma}$, $\upsilon_i \triangleq 1/\gamma_i$, γ_i is the path loss of the i-th BS-MS link and d_0 is one of the $m+1$ roots of the following polynomial

$$P_m(d) = (d\omega - \beta)\prod_{i=1}^{m}(d + \omega\upsilon_i) + d\alpha \sum_{i=1}^{m} \prod_{\substack{k=1 \\ k \neq i}}^{m} (d + \omega\upsilon_k) = 0. \tag{5}$$

Consequently, knowing that $C(\widetilde{\mathbf{H}})$ is equivalent to a Gaussian RV and applying (4) in (2), the outage probability of DMIMO systems can be well approximated by

$$P_{\text{out}} \approx Q\left(\frac{\mu_{C(\widetilde{\mathbf{H}})} - R}{\sigma_{C(\widetilde{\mathbf{H}})}}\right) \tag{6}$$

for large values of p, q and $m > 1$.

It can be noticed in (4) that both $\mu_{C(\widetilde{\mathbf{H}})}$ and $\sigma^2_{C(\widetilde{\mathbf{H}})}$ depend on the value of d_0. The root d_0 can be chosen amongst $m+1$ roots of $P_m(d)$, and hence the formula in (6) would provide an unique outage probability result only if the selection of d_0 is unique for any given $m > 1$. In the next section, we first prove the uniqueness of the selection of d_0 and express d_0 in function of the parameters $\omega, \upsilon_i, \alpha, \beta$ for $m = 2$ in Section 3.1. We then extend this proof for the case of $m \geq 2$ in Section 3.2.

3. APPROPRIATE SELECTION OF THE ROOT d_0

The selection of d_0 is constrained by the fact that $\mu_{C(\widetilde{\mathbf{H}})} \in \mathbb{R}$ and $\sigma^2_{C(\widetilde{\mathbf{H}})} > 0$ in (4). These two constraints imply that

$$\begin{cases} \mu_{C(\widetilde{\mathbf{H}})} \in \mathbb{R} \Leftrightarrow \ln\left(\frac{d_0}{\beta}\right) \in \mathbb{R} \Leftrightarrow d_0 \geq 0, \\ \sigma^2_{C(\widetilde{\mathbf{H}})} > 0 \Leftrightarrow 0 < \sum_{i=1}^{m}\left(\frac{d_0}{\upsilon_i\omega + d_0}\right)^2 \leq \frac{q}{p}. \end{cases} \tag{7}$$

Using $d_0 \geq 0$ in the second constraint of (7), we obtain after some simplifications that [13]

$$\min_i \left\{\frac{\omega\upsilon_i\sqrt{q/(mp)}}{1 - \sqrt{q/(mp)}}\right\} \geq d_0 \geq 0 \Rightarrow \sum_{i=1}^{m}\left(\frac{d_0}{d_0 + \omega\upsilon_i}\right)^2 \leq \frac{q}{p}. \tag{8}$$

Thus, the roots d_0 must be appropriately chosen amongst the $m+1$ root of $P_m(d)$ to fulfill the constraints set in (7) and (8).

3.1. DMIMO systems with two BSs

In the case of $m = 2$, the polynomial $P_m(d)$ in (5) can be simplified as $P_2(d) = c_3 d^3 + c_2 d^2 + c_1 d + c_0$, where the coefficients c_0, c_1, c_2, c_3 are given by

$$\begin{cases} c_1 = \omega[\omega^2 \upsilon_1 \upsilon_2 + (\upsilon_1 + \upsilon_2)(\alpha - \beta)], & c_0 = -\beta\omega^2 \upsilon_1 \upsilon_2, \\ c_2 = \omega^2(\upsilon_1 + \upsilon_2) + 2\alpha - \beta, & c_3 = \omega. \end{cases} \tag{9}$$

Note that $\omega \in \mathbb{R}_+$, since $\gamma \in \mathbb{R}_+$ and that $\upsilon_i \in \mathbb{R}_+$, since for any $i \in \{1, m\}$ $\gamma_i \in \mathbb{R}_+$. Furthermore, we consider that $p, q, n \geq 1$ and hence $\alpha, \beta \in \mathbb{Q}_+^*$, with $\mathbb{Q}_+^* = \{\frac{a}{b} | a \in \mathbb{N}, b \in \mathbb{N}, a, b \neq 0\}$, since $\alpha \triangleq p/n$ and $\beta \triangleq q/n$. In other words, we have $\omega, \upsilon_i \geq 0$, and $\alpha, \beta > 0$. The roots of $P_2(d)$ are then obtained by solving the cubic equation $P_2(d) = 0$ and its discriminant is usually expressed in terms of the coefficients c_0, c_1, c_2, c_3 as follows [14]

$$\Delta = \left(\frac{3c_1c_3 - c_2^2}{9c_3^2}\right)^3 + \left(\frac{9c_1c_2c_3 - 27c_0c_3^2 - 2c_2^3}{54c_3^3}\right)^2. \tag{10}$$

Inserting the values of c_0, c_1, c_2, c_3 in (10) and after intricate computations, it can be shown that $\Delta \leq 0$ in (10) for any $\omega, \upsilon_1, \upsilon_2 \geq 0$, and $\alpha, \beta > 0$ [13]. In this case, all the roots of $P_2(d)$ are real and can be expressed after some simplifications as follows [14]

$$d_k = -\frac{1}{3c_3}\left[\zeta_k\sqrt{c_2^2 - 3c_1c_3} + c_2\right], \tag{11}$$

where $\zeta_0 = -2\cos(\frac{\theta}{3})$, $\zeta_1 = 2\cos(\frac{\pi+\theta}{3})$, $\zeta_2 = 2\cos(\frac{\pi-\theta}{3})$, $k \in \{0, 2\}$, $\theta \triangleq \operatorname{acos}\left(\frac{9c_1c_2c_3 - 27c_0c_3^2 - 2c_2^3}{2(c_2^2 - 3c_1c_3)^{\frac{3}{2}}}\right)$, $\theta \in [0, \pi]$, and $\operatorname{acos} : [-1, 1] \mapsto [0, \pi]$ is the inverse cosine function. Using a proof by contradiction, we first show that the root d_0 in (11) is a real nonnegative root for any $\omega, \upsilon_1, \upsilon_2 \geq 0$, and $\alpha, \beta > 0$. Let us assume that $d_0 < 0$, it then implies with the definition of d_0 in (11) that

$$\frac{2\cos\left(\frac{\theta}{3}\right)\sqrt{c_2^2 - 3c_1c_3} - c_2}{3c_3} < 0. \tag{12}$$

Equivalently (12) can be re-expressed as

$$\cos\left(\frac{\theta}{3}\right) < \frac{c_2}{2\sqrt{c_2^2 - 3c_1c_3}}. \tag{13}$$

Knowing that $\frac{c_2}{2\sqrt{c_2^2 - 3c_1c_3}} \in [-1, 1]$, we apply the function acos to both sides of the inequality in (13), and this inequality simplifies as

$$\operatorname{acos}\left(\frac{9c_1c_2c_3 - 27c_0c_3^2 - 2c_2^3}{2(c_2^2 - 3c_1c_3)^{\frac{3}{2}}}\right) > 3\operatorname{acos}\left(\frac{c_2}{2\sqrt{c_2^2 - 3c_1c_3}}\right). \tag{14}$$

Notice that the relation in (13) is reversed in (14) because the function acos is a monotonically decreasing function. Next, we apply the function cosine, which is a monotonically decreasing function over $[0, \pi]$, to both sides of the inequality in (14) and re-expressed (14) as

$$\frac{9c_1c_2c_3 - 27c_0c_3^2 - 2c_2^3}{2(c_2^2 - 3c_1c_3)^{\frac{3}{2}}} < \frac{c_2}{2\sqrt{c_2^2 - 3c_1c_3}}\left(\frac{c_2^2}{c_2^2 - 3c_1c_3} - 3\right), \tag{15}$$

knowing that $\cos(3\operatorname{acos}(x)) = x(4x^2 - 3)$. Further derivation steps lead to

$$\begin{array}{rcl} 9c_1c_2c_3 - 27c_0c_3^2 - 2c_2^3 & < & c_2^3 - 3(c_2^3 - 3c_1c_2c_3), \\ -27c_0c_3^2 & < & 0. \end{array} \tag{16}$$

Using the definition of c_0, c_3 in (9), the proof can be summarized as $d_0 < 0 \Leftrightarrow 27\beta\omega^4\upsilon_1\upsilon_2 < 0$. However, we know that $\omega, \upsilon_1, \upsilon_2 \geq 0$ and $\beta > 0$, hence $27\beta\omega^4\upsilon_1\upsilon_2 \geq 0$, which contradicts the result of the proof, and consequently d_0 is a real nonnegative root, i.e., $d_0 \in \mathbb{R}_+$. Moreover, using a similar approach for d_1 by assuming that $d_1 > 0$ with

$$\left(-2\cos\left(\frac{\theta + \pi}{3}\right)\sqrt{c_2^2 - 3c_1c_3} - c_2\right)/(3c_3) > 0. \tag{17}$$

as a starting point, we obtain after some simplifications that (17) is equivalent to $-27c_0c_3^2 < 0$, i.e., $27\beta\omega^4\upsilon_1\upsilon_2 < 0$ [13]. Thus, the result of the proof contradicts the aforementioned assumption and, hence, $d_1 \leq 0$. Finally, the parameters ζ_k in (11) can clearly be ordered as follows $\zeta_0 \leq \zeta_1 \leq \zeta_2$, since $\zeta_0 \in [-2, -1]$, $\zeta_1 \in [-1, 1]$ and $\zeta_2 \in [1, 2]$. This ordering straightforwardly indicates that $d_2 \leq d_1 \leq 0 \leq d_0$.

Therefore, $P_2(d) = 0$ has only one real nonnegative solution, which is denoted d_0 and is expressed in terms of ω, υ_1, υ_2, α and β as follows

$$d_0 = \frac{1}{3\omega}\left[2\cos\left(\frac{\theta}{3}\right)\left[\omega^4(\upsilon_1^2 - \upsilon_1\upsilon_2 + \upsilon_2^2) + \omega^2(\upsilon_1 + \upsilon_2) \times(\alpha + \beta) + (2\alpha - \beta)^2\right]^{\frac{1}{2}} - \omega^2(\upsilon_1 + \upsilon_2) - (2\alpha - \beta)\right], \tag{18}$$

where θ is given at the bottom of the page. Note that d_0 in (18) is the only root of $P_2(d)$ that fulfils the constraints set in (7) and (8). Hence, only d_0 can be used in (4) to compute the closed-form approximation in (6) when $m = 2$.

$$\theta = \operatorname{acos}\left(-\frac{2\omega^6(\upsilon_1 + \upsilon_2)(\upsilon_1 - 2\upsilon_2)(\upsilon_1 - 0.5\upsilon_2) + 3\omega^4(\upsilon_1^2 - 4\upsilon_1\upsilon_2 + \upsilon_2^2)(\alpha + \beta) + 3\omega^2(\upsilon_1 + \upsilon_2)(\alpha + \beta)(2\alpha - \beta) + 2(2\alpha - \beta)^3}{2\left[\omega^4(\upsilon_1^2 - \upsilon_1\upsilon_2 + \upsilon_2^2) + \omega^2(\upsilon_1 + \upsilon_2)(\alpha + \beta) + (2\alpha - \beta)^2\right]^{\frac{3}{2}}}\right)$$

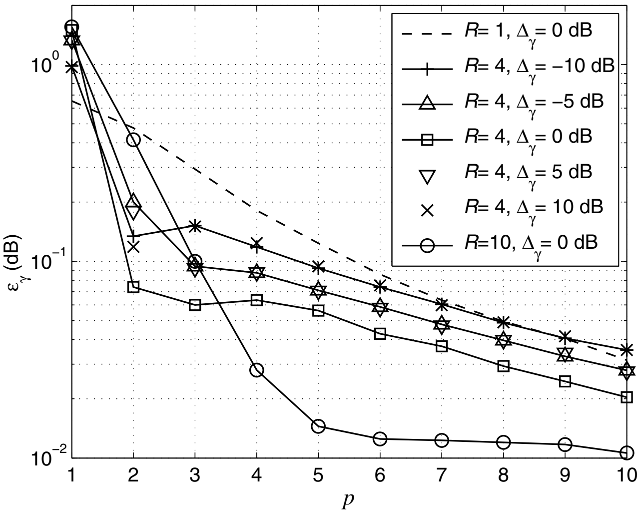

Fig. 2. Accuracy of (6) against the number of antennas for a two-BSs DMIMO system.

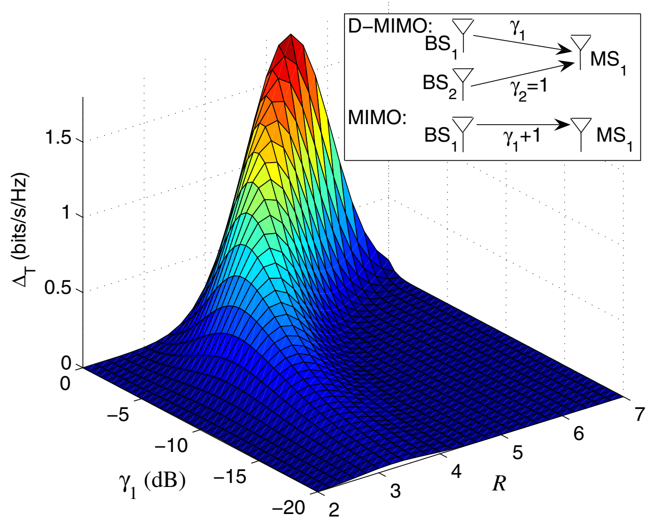

Fig. 3. Theoretical throughput comparison of a MIMO system against a two-BSs DMIMO system.

3.2. DMIMO systems with $m > 2$ BSs

In the general case, the polynomial $P_m(d)$ in (5) can be simplified as $P_m(d) = \sum_{i=0}^{m+1} c_i d^i$, where the coefficients c_i are given by

$$c_i = \begin{cases} \omega & \text{, if } i = m+1, \\ \omega^{m-i}(\omega^2 a_{m,i} + (i\alpha - \beta) a_{m,i+1}) & \text{, else,} \end{cases} \tag{19}$$

where

$$a_{k,j} = \begin{cases} 0 & \text{, if } j = 0, k \in \{0, m\}, \\ 1 & \text{, if } j = k+1, k \in \{0, m\}, \\ a_{k-1,j}\upsilon_k + a_{k-1,j-1} & \text{, else.} \end{cases} \tag{20}$$

In the case that $\upsilon_1 \neq \upsilon_2$ and $\upsilon_k = \upsilon_2$ for $k \in \{3, m\}$, it can be shown that [13]

$$P_m(d) = (d + \omega\upsilon_2)^{m-2}[P_2(d) + (m-2)\alpha d(d + \omega\upsilon_1)], \tag{21}$$

for $m \geq 3$. On the one hand, the roots $d_3, \ldots, d_m$ are all equal to $-w\upsilon_2$ and are thus strictly negative. On the other hand, the coefficients c_2 and c_1 in (9) are modified as $c_2' = c_2 + (m-2)\alpha$ and $c_1' = c_1 + w(m-2)\alpha$. Inserting the values of c_2' and c_1' instead of c_2 and c_1 in (9), will not affect that $d_0 \geq 0$, $d_1 \leq 0$ and $d_2 \leq 0$. Therefore, d_0 will still be the unique nonnegative roots of $P_m(d)$, and it can be recomputed using (11) with $c_2 = c_2'$ and $c_1 = c_1'$.

Furthermore, in the case that $\upsilon_j \neq \upsilon_k$, $j \neq k$, $j, k \in \{1, m\}$, d_0 is still the only nonnegative root of $P_m(d)$, as explained in [15].

4. NUMERICAL RESULTS AND APPLICATIONS FOR P_{OUT}

Our closed-form approximation in (6) can be used to evaluate and compare the outage probability of DMIMO systems faster than time consuming Monte-Carlo simulations. However, the accuracy of this approximation depends on the values of m, p and q. In Fig. 2, we establish numerically for a DMIMO system with two BSs, i.e., $m = 2$, the values of p and q for which our closed-form approximation in (6) has a sufficient accuracy. We define γ_{TH} and γ_{MC} as the theoretical SNR and the SNR obtained via Monte-Carlo simulation, respectively, which are required to achieve a certain rate R for a given outage probability P_{out}. The absolute difference in dB between these two SNRs, i.e., $\varepsilon_\gamma(\text{dB}) \triangleq |\gamma_{TH}(\text{dB}) - \gamma_{MC}(\text{dB})|$, provides a reliable measure of the accuracy of (6). The results in Fig. 2 has been obtained using 1×10^6 channel realizations for the Monte-Carlo simulation. Concerning the two-BSs DMIMO system, we introduce $\Delta_\gamma(\text{dB}) \triangleq \gamma_2(\text{dB}) - \gamma_1(\text{dB})$ as the SNR offset between the two BSs resulting from unequal path losses and shadowing between each BS and the MS. In order to evaluate (6) for $m = 2$, we use d_0 in (18).

In Fig. 2, $\varepsilon_\gamma(\text{dB})$ is plotted against the number of transmit antennas p, using a two-BSs DMIMO system, for $P_{\text{out}} = 1 \times 10^{-2}$, $\beta = 1$, i.e., $q = p$, various rates R, several values of $\Delta_\gamma(\text{dB})$, and the downlink case. The results confirm that the instantaneous capacity $C(\mathbf{H})$ becomes more equivalent to a Gaussian RV as the values of p grows larger, since $\varepsilon_\gamma(\text{dB})$ decreases with p. Thus, the accuracy of our closed-form approximation in (6) increases in this case. The result also indicates that the accuracy of (6) decreases when $|\Delta_\gamma|(\text{dB})$ increases. Moreover, we observe that for $\beta = 1$ and $p \geq 4$, (6) provides a sufficient accuracy for any rate, i.e., $\varepsilon_\gamma(\text{dB}) < 0.2$ dB. Notice that similar results have been obtained for the uplink case. Finally, (6) being an approximation and not a bound, the theoretical and Monte-carlo curves can criss-cross and this explain the fluctuations of ε_γ(dB) for low p values.

Assuming an unlimited number of ARQs, the theoretical throughput of communication systems can be expressed as $T = (1 - P_{\text{out}})R$. Using (6) to evaluate P_{out}, we plot in Fig. 3 the difference Δ_T between the theoretical throughput of a MIMO system and a two-BSs DMIMO system in function of the rate R and of the SNR γ_1(dB), for $p = q = 4$, $\gamma = 1$, and the downlink case. Moreover as indicated in the top right corner of Fig. 3, we fixed $\gamma_2 = 1$ for the DMIMO system and we considered an aggregate SNR $\gamma_1 + 1$ for the MIMO system, in order to fairly compare the two systems for the same total power. Results in Fig. 3 show that a throughput gain resulting from spatial diversity can be achieved by using a DMIMO system instead of a MIMO system. In Fig. 3, a maximum gain of 1.8 bits/s/Hz is reached for $\gamma_1 = 0$ dB and a rate of $R \simeq 4.3$ bits/s/Hz.

5. CONCLUSION

In this paper, a closed-form approximation of the outage probability for DMIMO systems has been derived for both downlink and uplink cases. It is shown that the instantaneous capacity of DMIMO systems over Rayleigh fading channel is asymptotically equivalent to a Gaussian RV and the mean and variance of the equivalent Gaussian RV are given in terms of the roots of a degree-$(m+1)$ polynomial. The uniqueness of our closed-form approximation is then related to the selection of one root amongst $m + 1$ possible roots. The uniqueness of the root selection has been firstly proved for $m = 2$ and the expression of the root itself has been provided for this case. Then, the proof has been extended for the case of $m \geq 2$. Furthermore, the accuracy of our closed-form approximation has been assessed for different SNR settings, rates, and number of antennas. Results have indicated that the accuracy of our approximation is sufficient for $p = 4$ when $m = 2$. Finally, our expression has been utilized to show that DMIMO systems can achieve higher throughput than MIMO systems by using spatial diversity. In the future, we will consider a less idealized base-station cooperative model by taking into account the erroneous communications between the base stations as well as the bandwidth loss due to such communications.

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